Nuclear Forces from Lattice QCD

T. Hatsuda (Univ. Tokyo)

HAL QCD Collaboration
(Hadrons to Atomic Nuclei Lattice QCD Collaboration)

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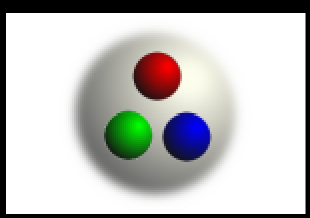
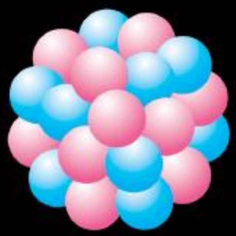
T. Hatsuda, Y. Ikeda, N. Ishii (Univ. Tokyo)
H. Nemura (Tohoku Univ.)

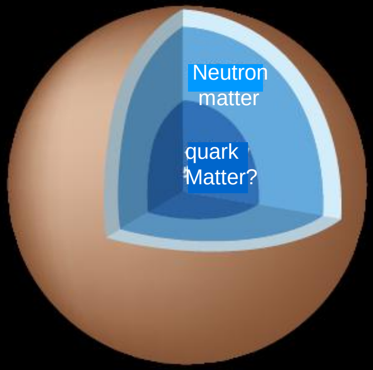


Hadrons

NN, YN, YY, 3N
forces from LQCD



Atomic nuclei



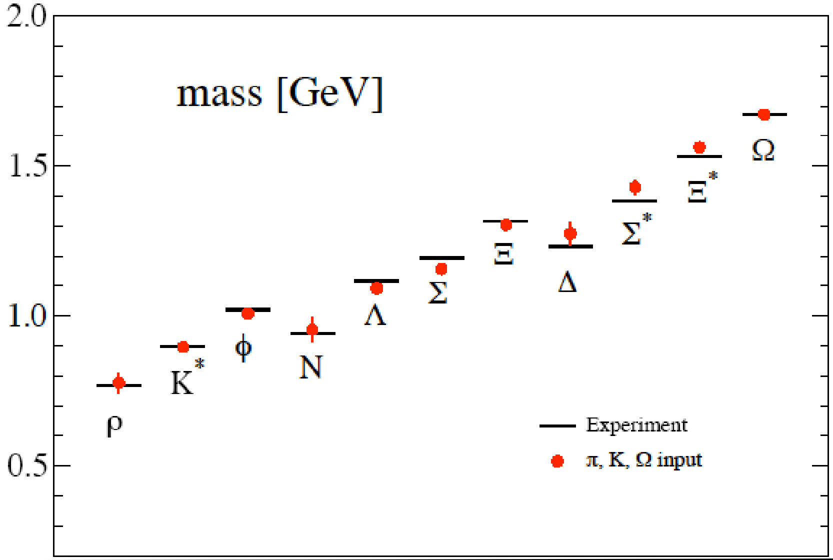
Neutron star

Chiral Dynamics 09, Bern, July 7, 2009

Hadron masses on the lattice

PACS-CS gauge configurations(2+1 flavors)

Phys. Rev. D79(2009)034503

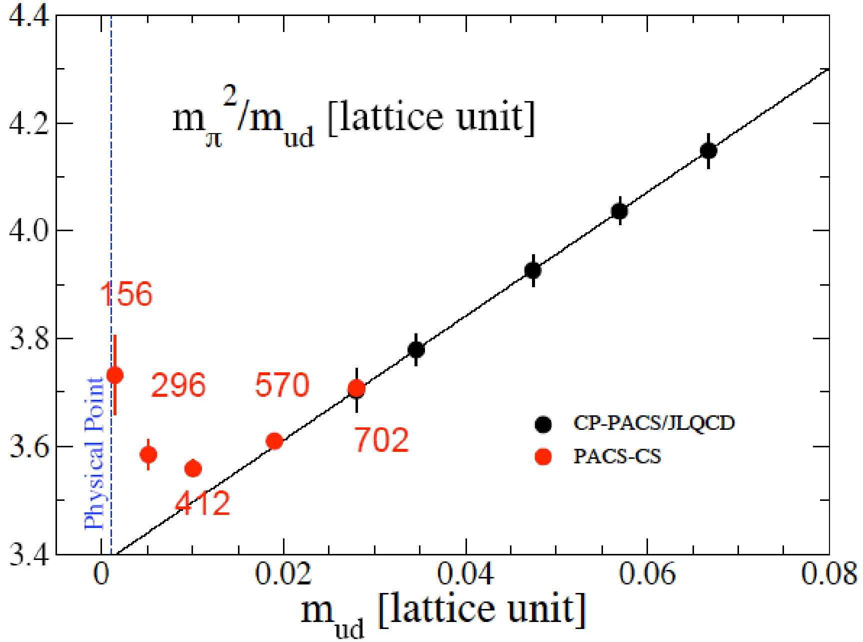


$$a = 0.09 \text{ fm}$$

$$L = 2.9 \text{ fm}$$

$$m_\pi^{\text{min.}} = 156 \text{ MeV} \qquad m_\pi L = 2.3$$

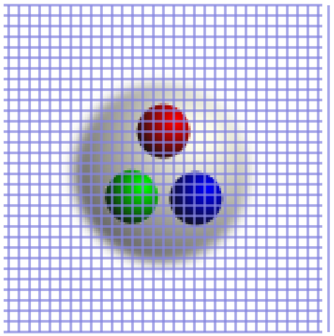
We are almost on the "physical point".



$$m_\pi L > 4$$

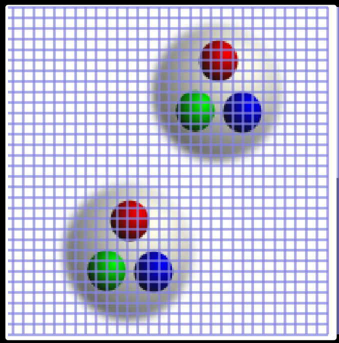
Calculations with L=5.8 fm and $m_\pi \simeq 140$ MeV are on-going.

"Real QCD"

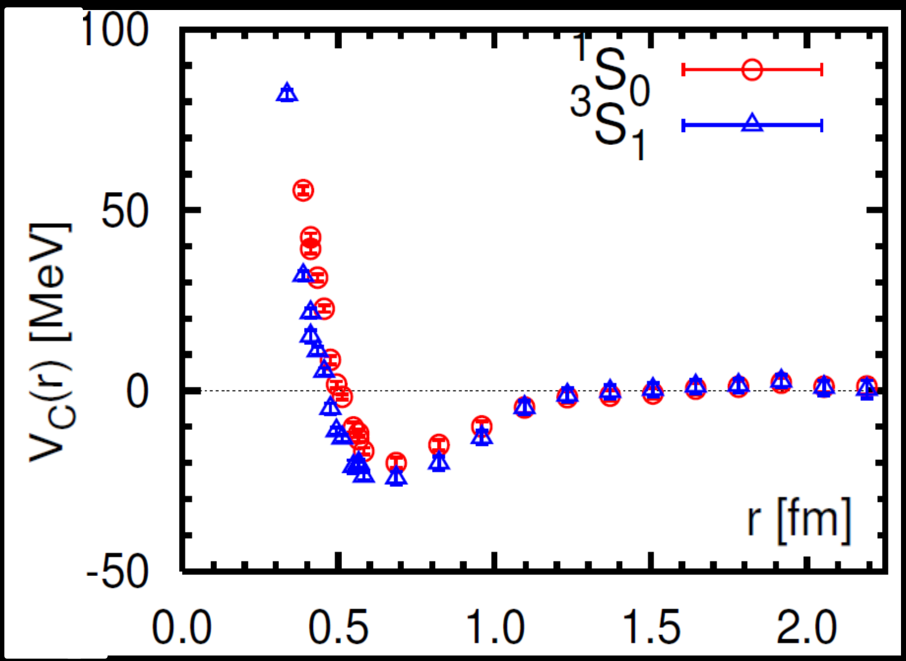


NN interaction on the lattice

Wave function
↓
NN potential

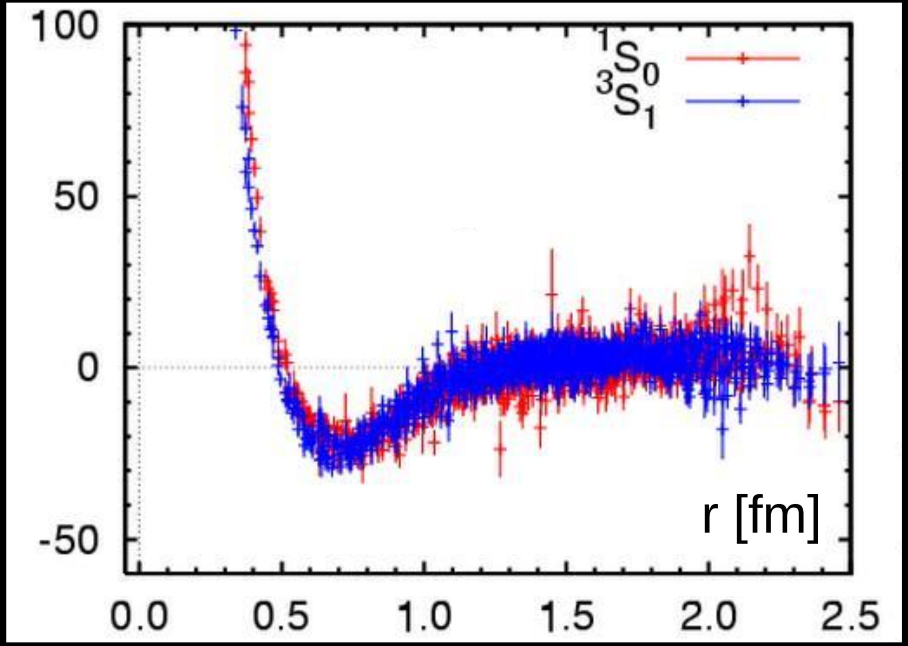


Quenched QCD
(m_π=530MeV, L=4.4 fm)



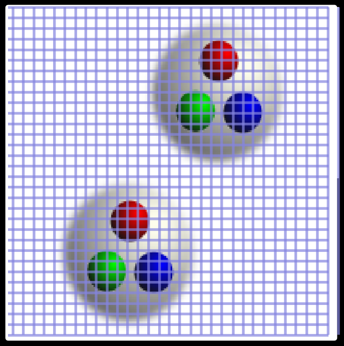
Ishii, Aoki & Hatsuda,
PRL 99 (2007) 022001

Full QCD
(m_π=570MeV, L=2.9 fm)



Ishii, Aoki & Hatsuda,
arXive 0903.5497 [hep-lat]

Methods to extract NN interaction from LQCD



Luscher, Nucl. Phys. B354 (1991) 531

[1] Temporal correlation : $E_{NN}(L)$ → NN phase shift

$$\frac{2Z_{00}(1,q)}{L\pi^{1/2}} = k \cot \delta_0(k)$$

- quenched QCD: CP-PACS Coll. (1995)
- full QCD: NPLQCD Coll. (2006-)

[2] Spatial correlation :
BS wave function → NN potential → observables

$$(E - H_0)\phi(\mathbf{r}) = \int U(\mathbf{r},\mathbf{r}')\phi(\mathbf{r}')d\mathbf{r}'$$

← half off-shell T-matrix

- π-π system : CP-PACS Coll. (2005)
- NN system (quenched QCD) : Ishii, Aoki & T.H., PRL 99, 022001 (2007).
- NN, YN systems (full QCD): HAL QCD Coll. (2008-)

HAL procedure

Ishii, Aoki & T.H., (2007, 2008)

(i) Take your favorite interpolating operator

e.g. $N(x) = \epsilon_{abc} q^a(x) q^b(x) q^c(x)$

← observables do not depend on the choice   Haag, Nishijima, Zimmermann (1958)

(ii) Calculate the equal-time BS amplitude

$$\phi(\vec{r}) = \langle 0 | N(\vec{x}+\vec{r}) N(\vec{x}) | 6q \rangle$$

(iii) Define the potential

$$(E - H_0)\phi(\vec{r}) = \int U(r, \vec{r}')\phi(\vec{r}') d^3 r'$$

(iv) Derivative expansion $U(\vec{r}, \vec{r'}) = V(\vec{r}, \nabla)\delta^3(\vec{r} - \vec{r}')$

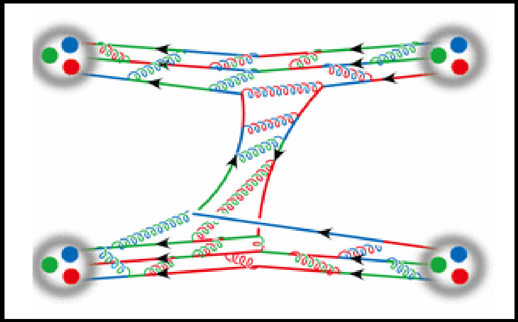
$$V(\vec{r}, \nabla) = V_{\rm C}(r) + S_{12} V_{\rm T}(r) + \vec{\rm L}\cdot\vec{\rm S}\ V_{\rm LS}(r) + \{V_{\rm D}(r), \nabla^2\} + \cdots$$

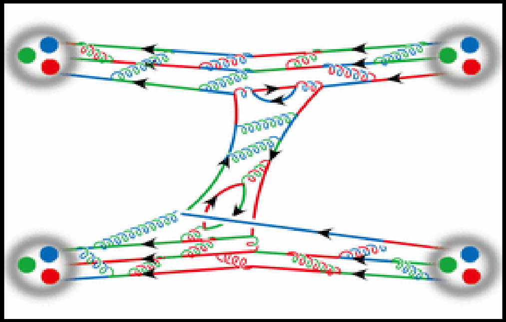
Okubo-Marshak (1958)

← successive determination using BS amplitudes for different E

→ calculate observables (phase shifts, binding energies etc)

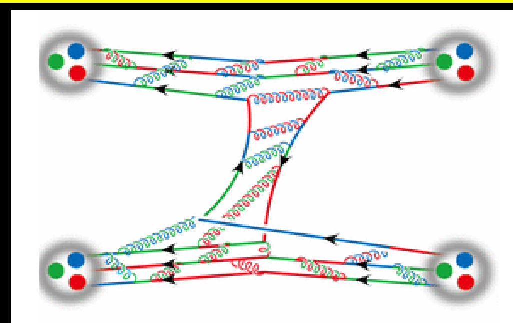
HAL setup

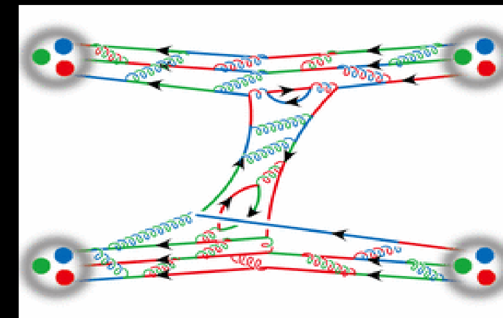




	Quenched QCD	Full QCD (N_f=2+1)
Configurations	BlueGene/L@KEK **(Iwasaki)** **# of config. ~ 2000**	PACS-CS@Tsukuba **(Iwasaki, Wilson+clover)** **# of config. ~ 500**
L (spatial size)	4.4 fm	2.9 fm
a (lattice spacing)	0.14 fm	0.091 fm
m_π	380 MeV, 529 MeV, 731 MeV	301 MeV, 415 MeV, 568 MeV, 700 MeV

Exploratory study in quenched QCD





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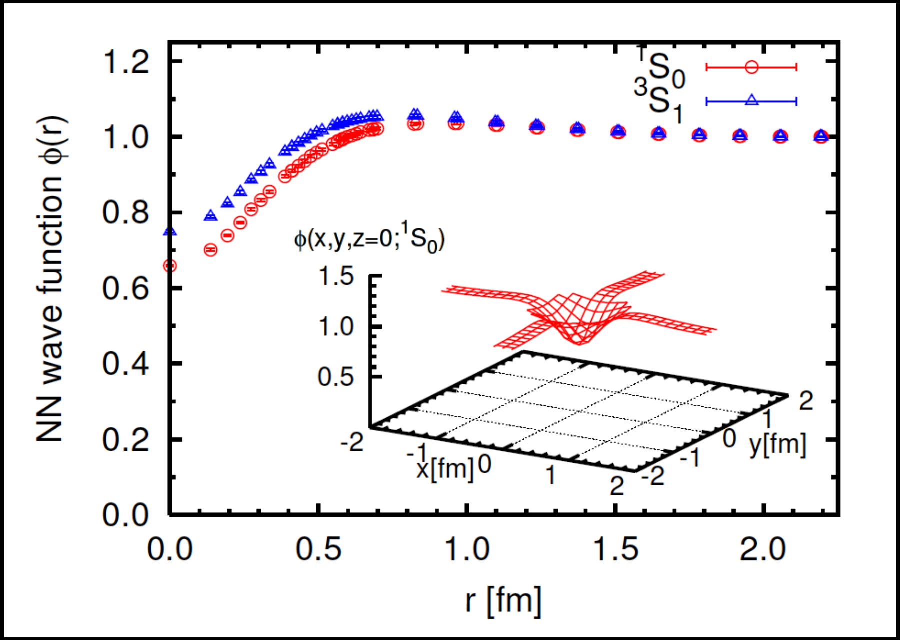
Central potential $V_c(r)$ from ϕ (**r**) at E ~ 0 (m_π =0.53 GeV)

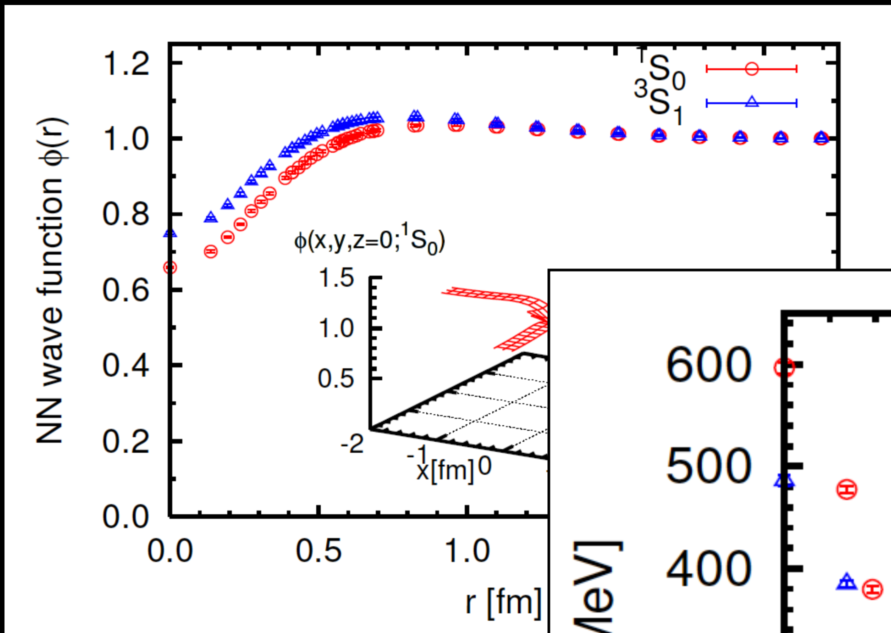
$^1S_0$,$^3S_1$



Equal-time BS amplitude

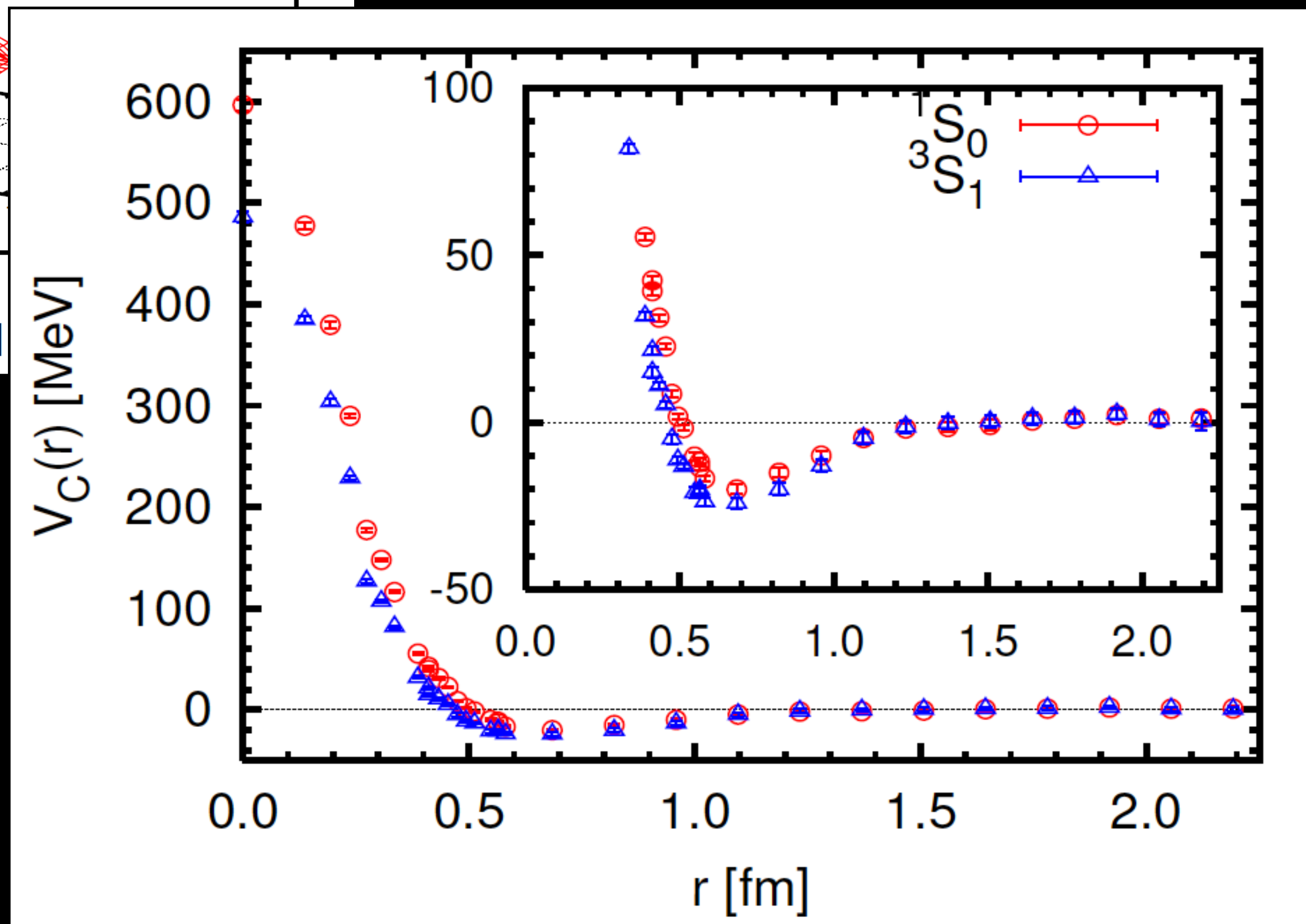
Central potential $V_c(r)$ from $\phi(\mathbf{r})$ at E ~ 0
(m_π =0.53 GeV)

$^1S_0$, $^3S_1$



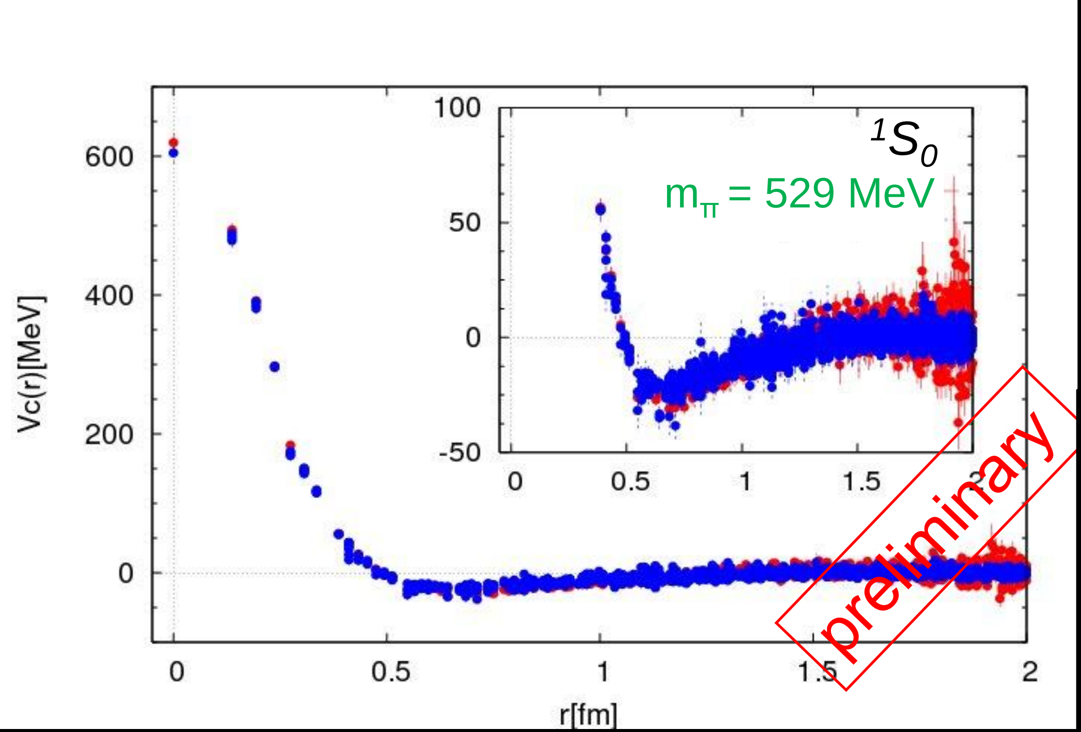
Equal-time BS amplitude

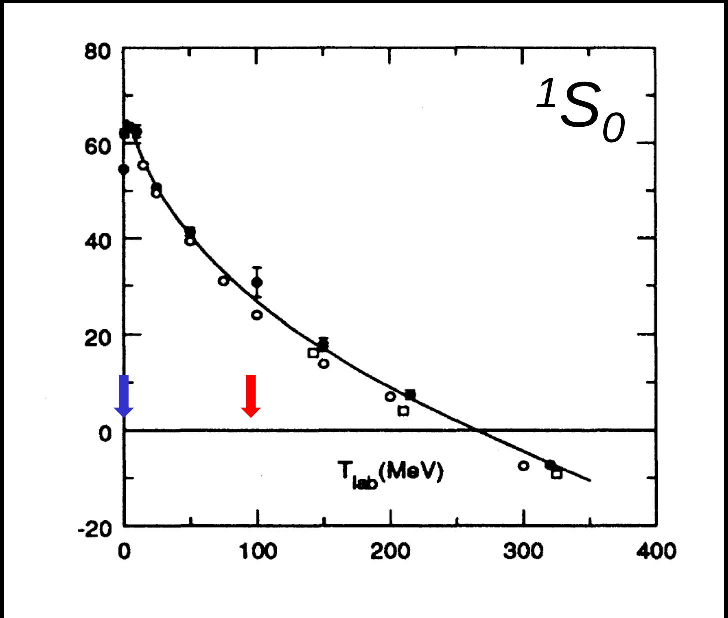
Central potential



velocity dependence of V

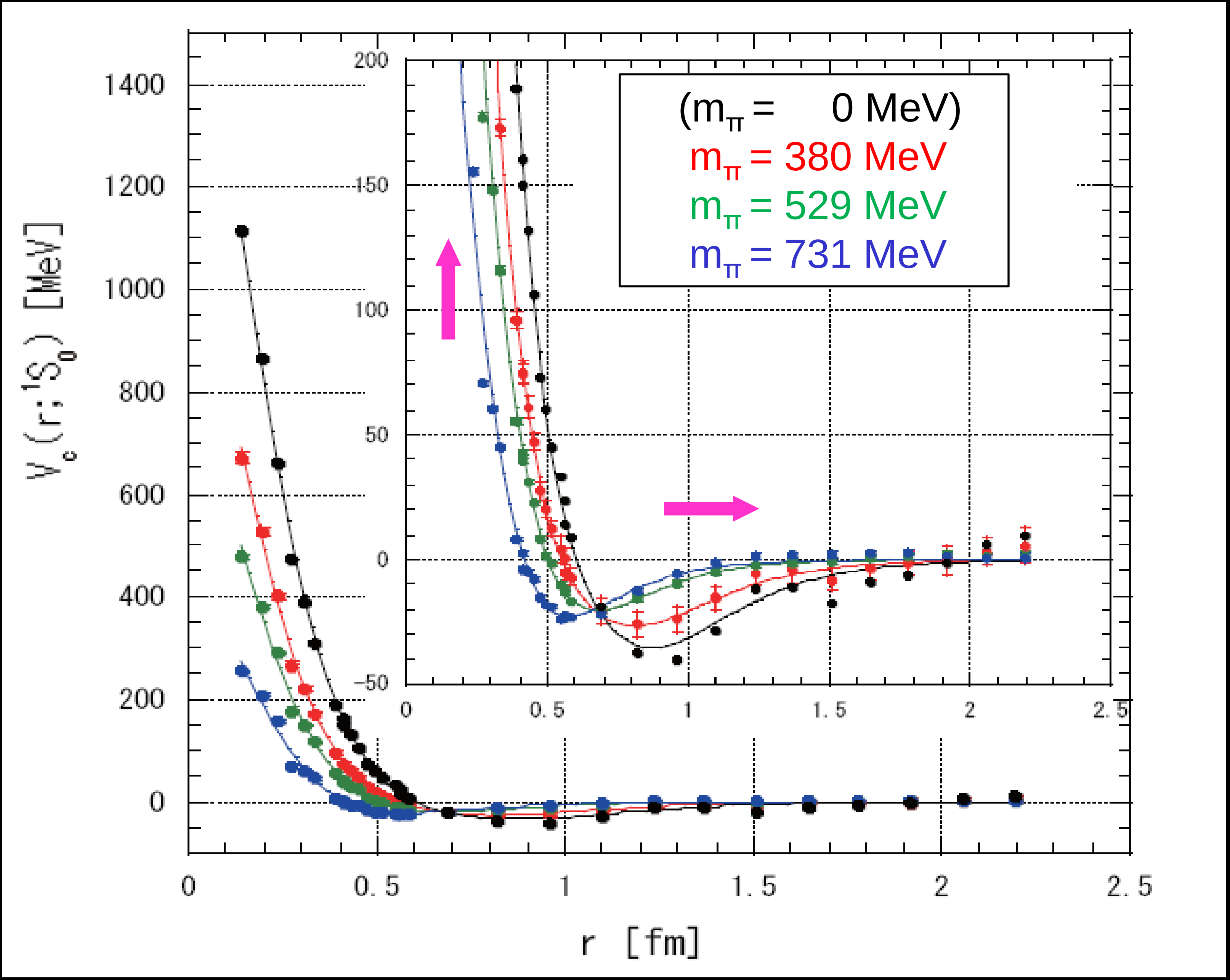
$$V(\vec{r},\nabla) = V_{\rm C}(r) + S_{12}V_{\rm T}(r) + \vec{\rm L}\cdot\vec{\rm S}\;V_{\rm LS}(r) + \{V_{\rm D}(r),\nabla^2\} + \cdots$$



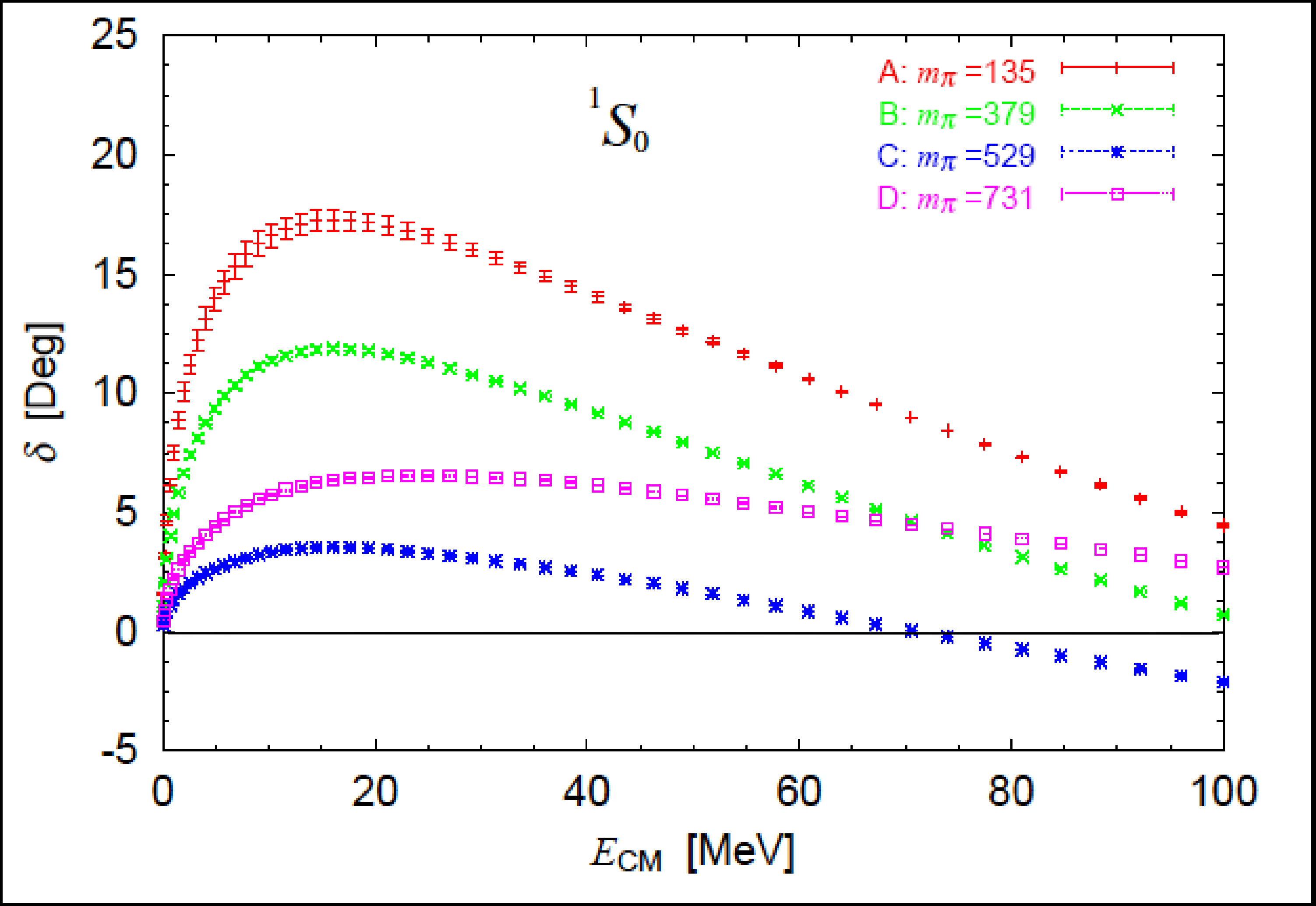


1. velocity-dep. terms can be determined from E-dependence of φ (r)
2. E-dep. turns out to be small at low energies in our choice of N(x)

Quark mass dependence of $V_c(r)$ in $^1S_0$

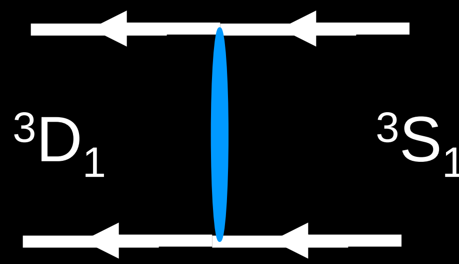


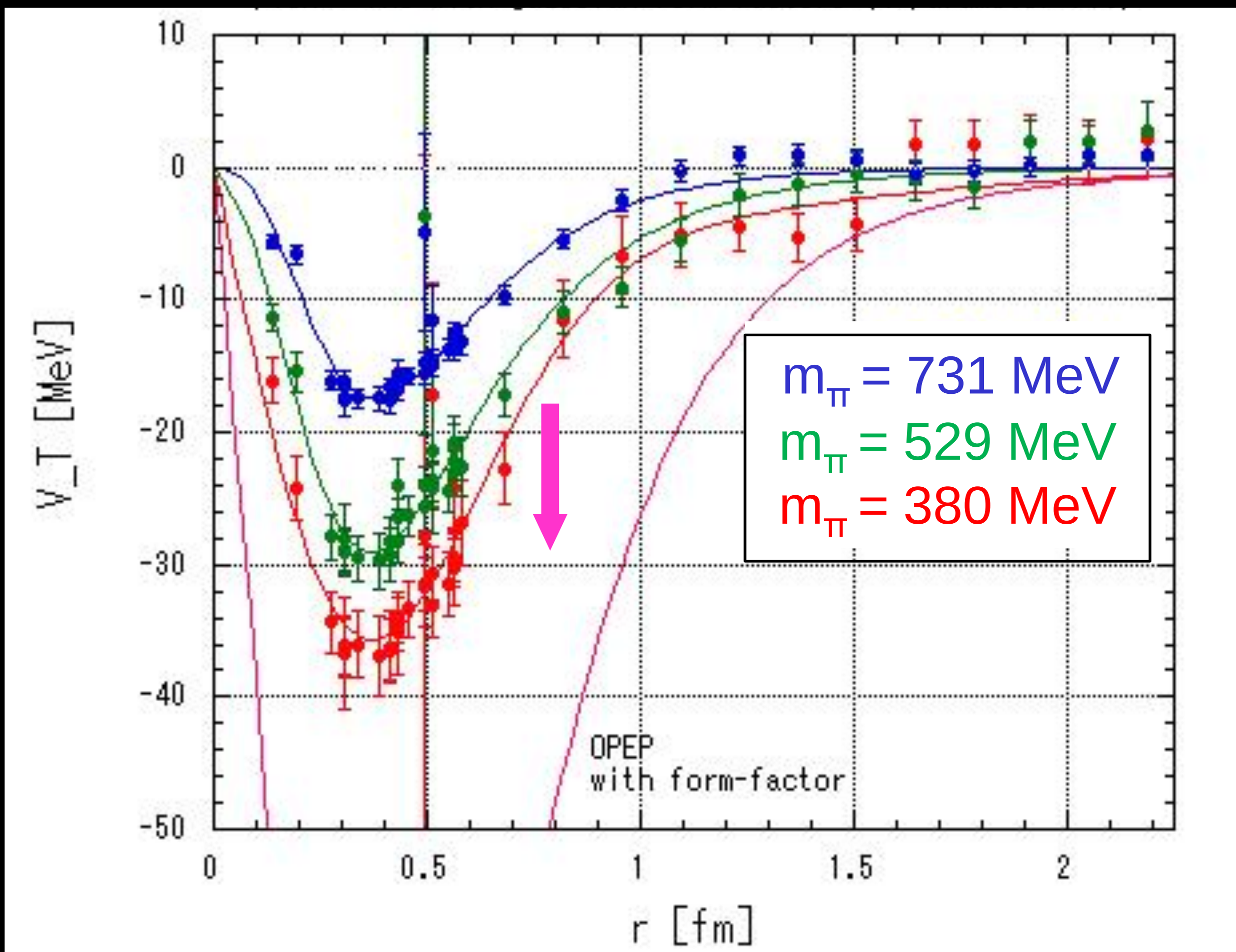
$^1S_0$ phase shift from $V_c(r)$



NN tensor force $V_T(r)$ and its quark-mass dependence

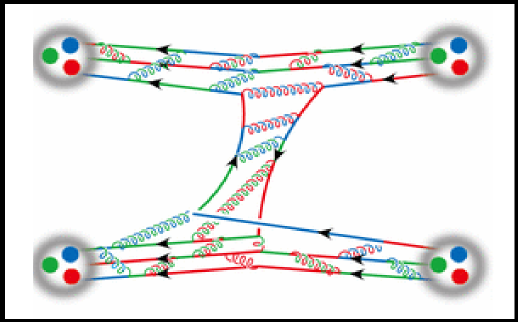
$$\left[-\frac{1}{2\mu}\vec{\nabla}^2+V_C(\vec{r})+V_T(\vec{r})S_{12}\right]\begin{pmatrix}\phi(\vec{r};{}^3S_1)\\ \phi(\vec{r};{}^3D_1)\end{pmatrix}=E\begin{pmatrix}\phi(\vec{r};{}^3S_1)\\ \phi(\vec{r};{}^3D_1)\end{pmatrix}$$

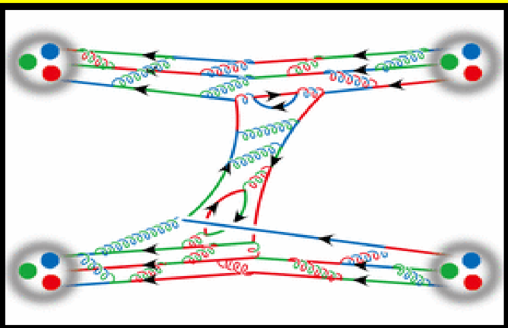




fit: π+ρ with gaussian form-factors

Full QCD





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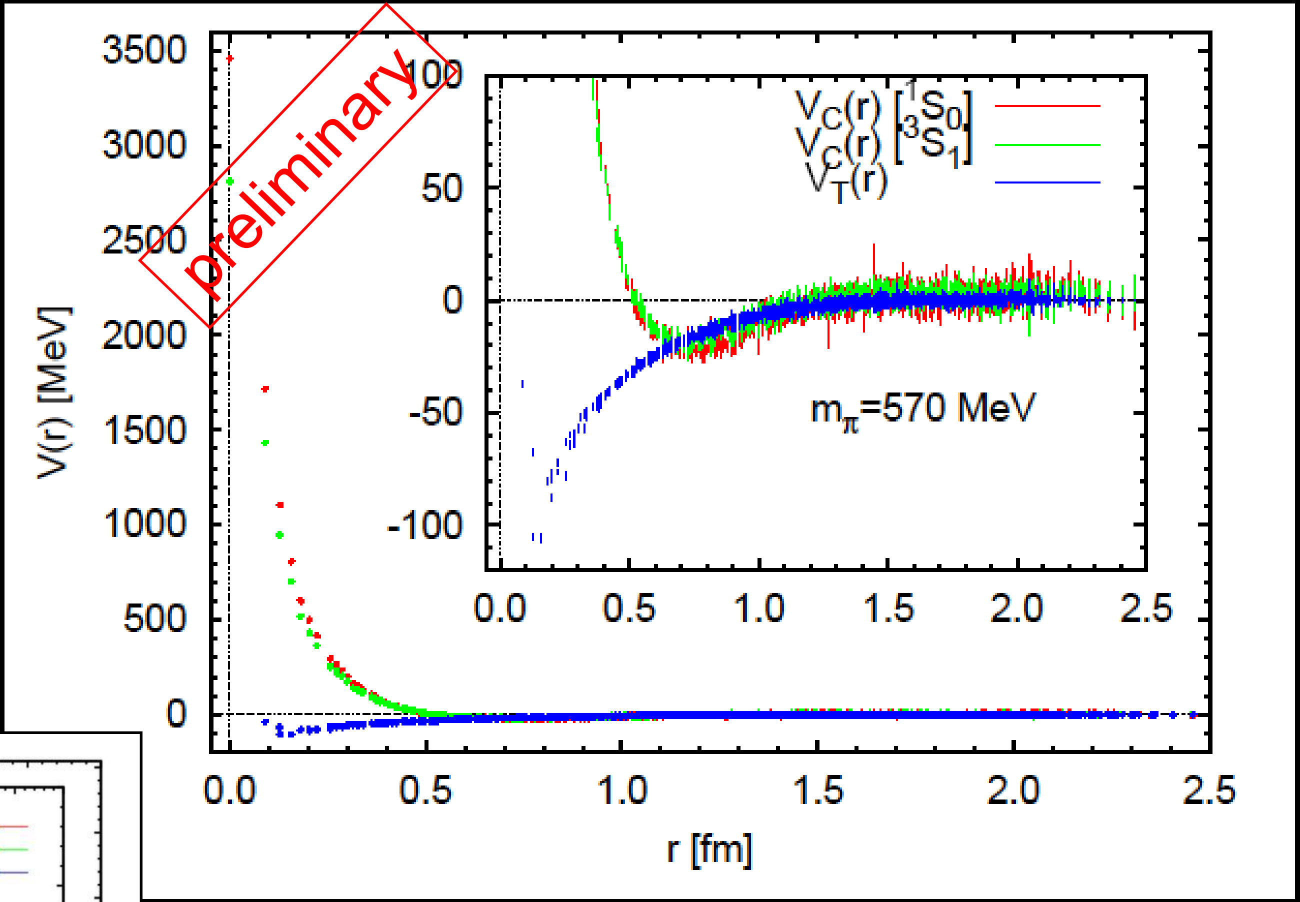
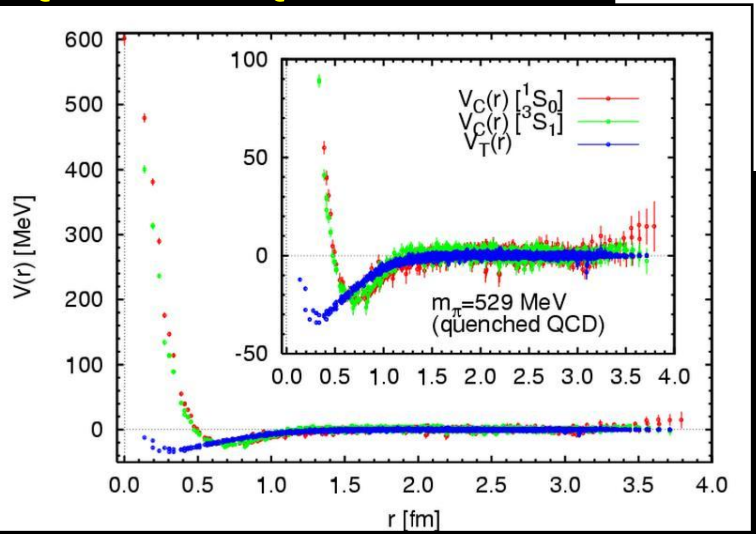
$V_C(r)$ and $V_T(r)$ in full QCD (m_π=570MeV, L=2.9 fm)



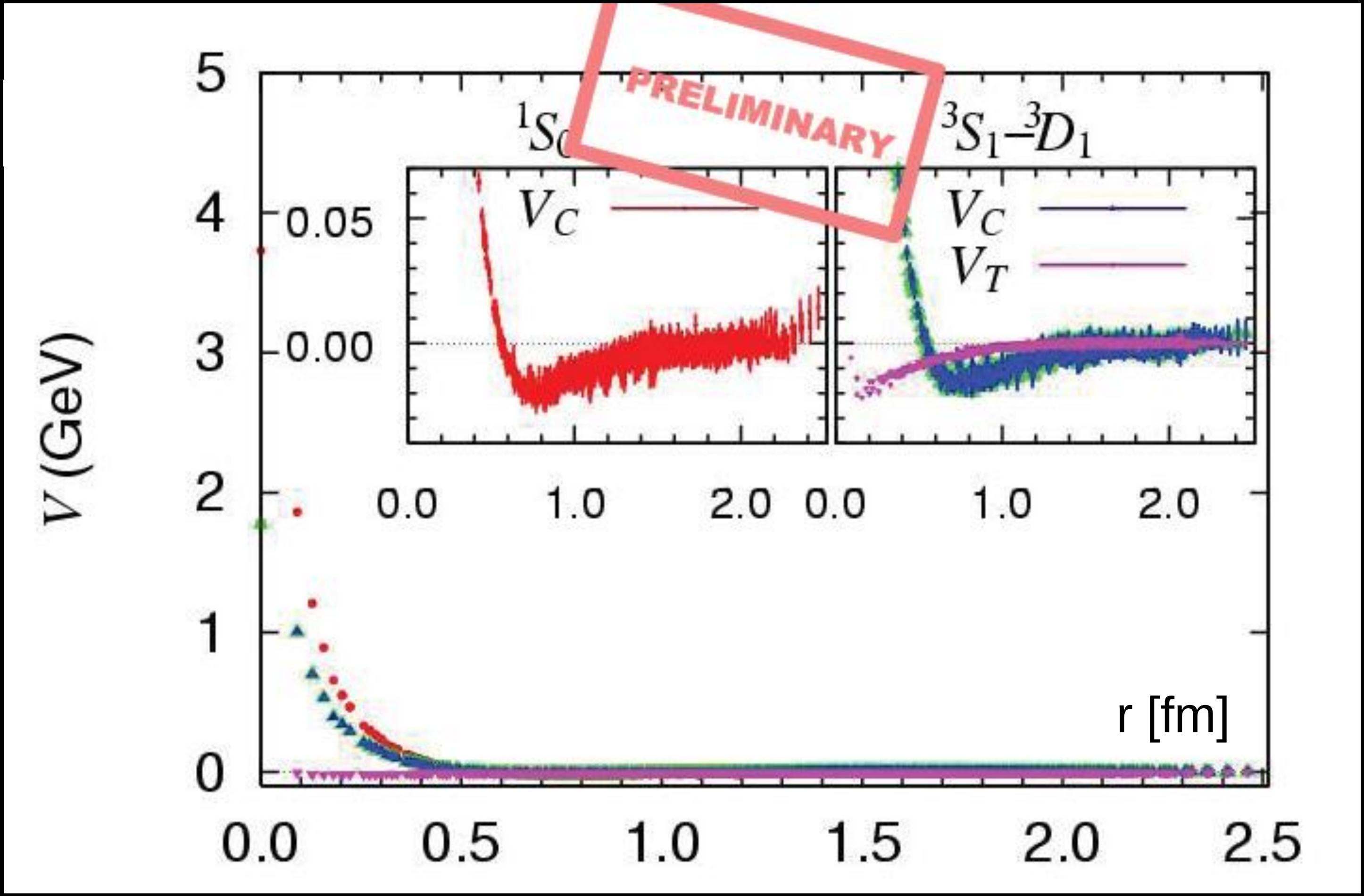
Full QCD

Quenched QCD



- Larger repulsive core than quenched
- Larger tensor force than quenched

ΛN in full QCD (m_π=415 MeV, L=2.9 fm)

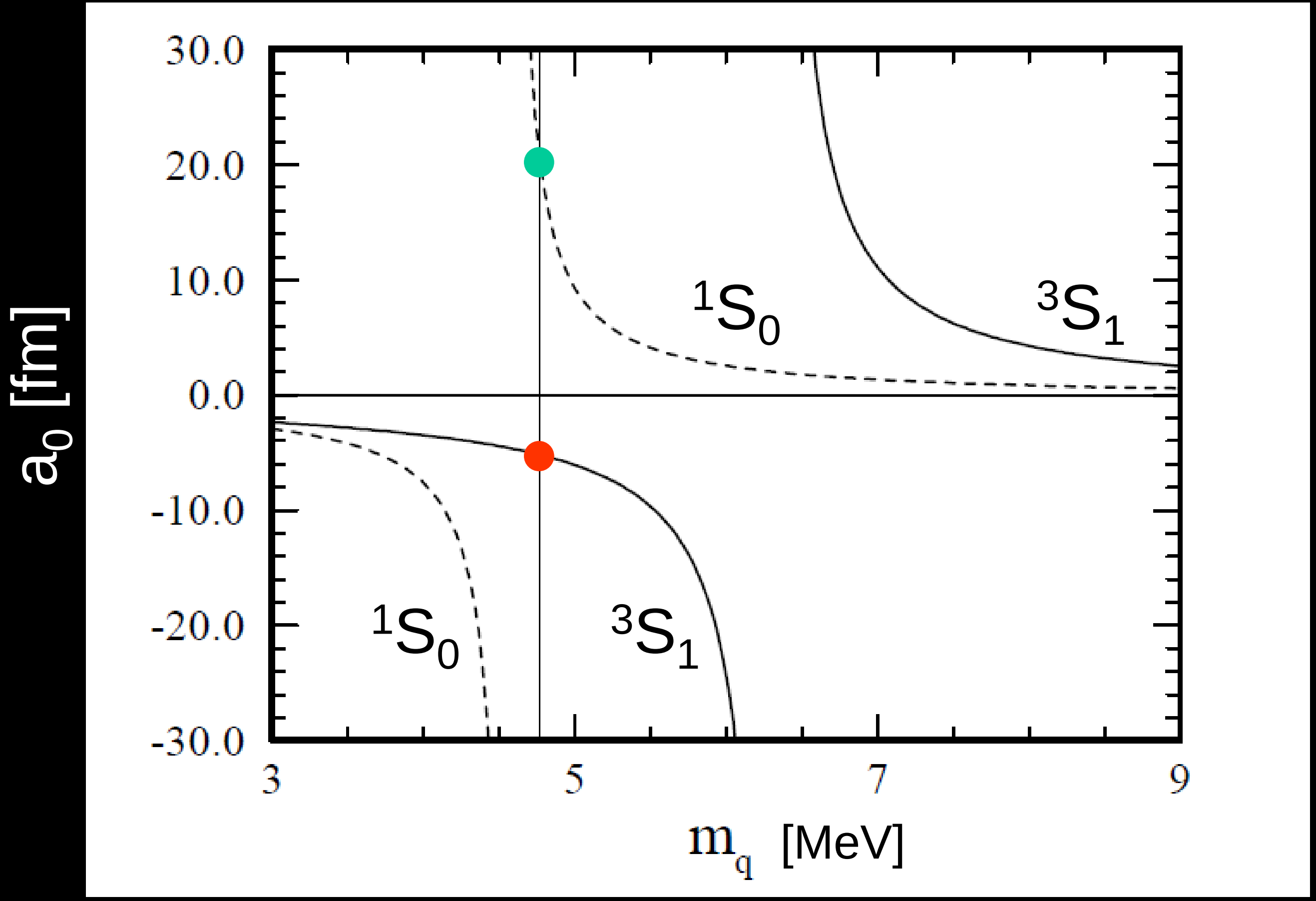


- Weaker repulsive core than NN
- Stronger spin-dependence than NN
- Weaker tensor force than NN

NN scattering length (Kuramashi plot)

Kuramashi, Prog. Theor. Phys. Suppl. 122 (1996) 153 [hep-lat/9510025]

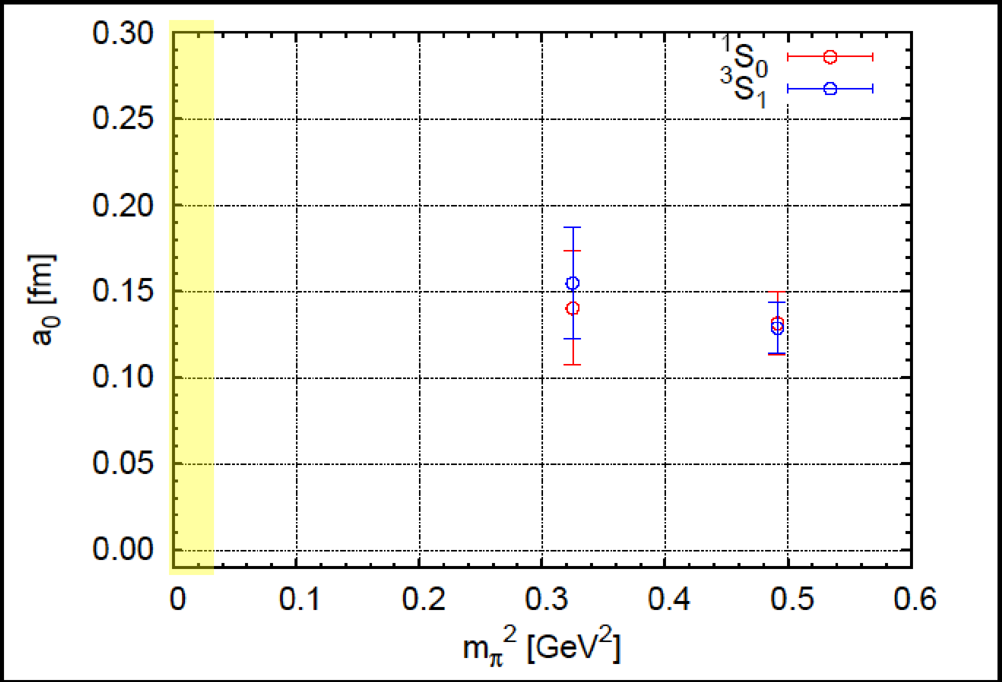
OBEP + lattice hadron mass



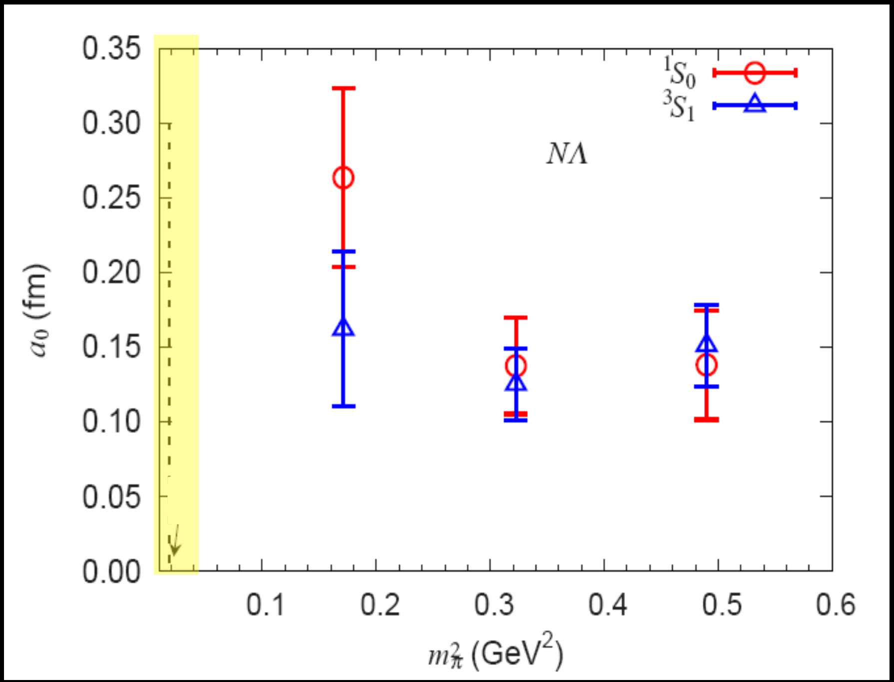
← Narrow unitary region →

Scattering lengths in full QCD

NN



ΛN



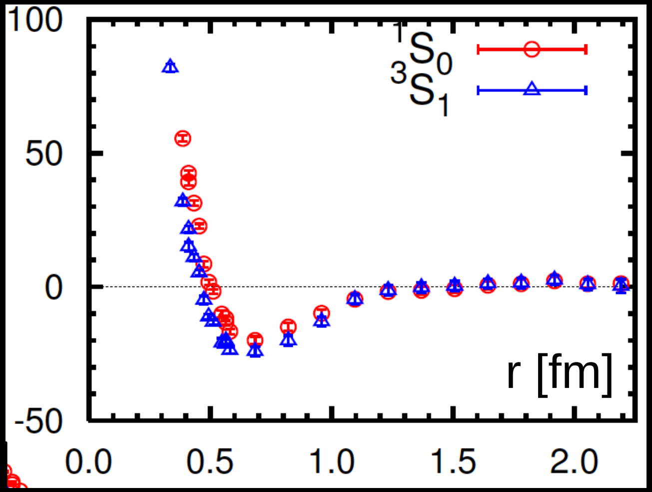
Summary

1. Nuclear force from LQCD
   - BS amplitude → NN, YN, YY potentials → observables

2. NN force in quenched QCD : good “shape”
   - repulsive core, intermediate attraction, tensor force

3. Hyperon forces :
   - ΞN, ΛN, ΣN, ΛΛ undrerway

     → inputs to hyper nuclear physics



Current and Future

○ Full QCD with m_π=140 MeV is our ultimate goal

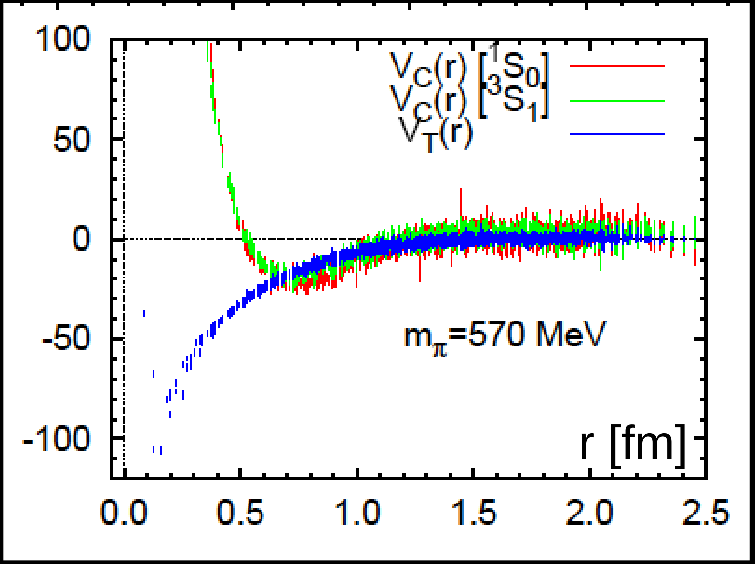
current : PACS-CS config. (N_f=2+1) with L=2.9fm & m_π = 156-701 MeV

in 1-2 years: PACS-CS config. (N_f=2+1) with L=5.8fm & m_π = 140 MeV

in 5 years: new config. on 20 Pflops machine (2011-)

○ Current and Future targets of HAL QCD Coll.

- tensor force and deuteron binding
- origin of the repulsive core
- LS force
- YN and YY forces
- 3N forces
- light nuclei from lattice QCD inputs
- relation to EFT



Some References

- NN force in quenched QCD:
  Ishii, Aoki & T.H., Phys. Rev. Lett. 99 (2007) 022001 [nucl-th/0611.096].

- Introductory review:
  Aoki, T.H. & Ishii, Comput. Sci. Disc. 1 (2008) 015009 [arXiv:0805.2462 [hep-ph]].

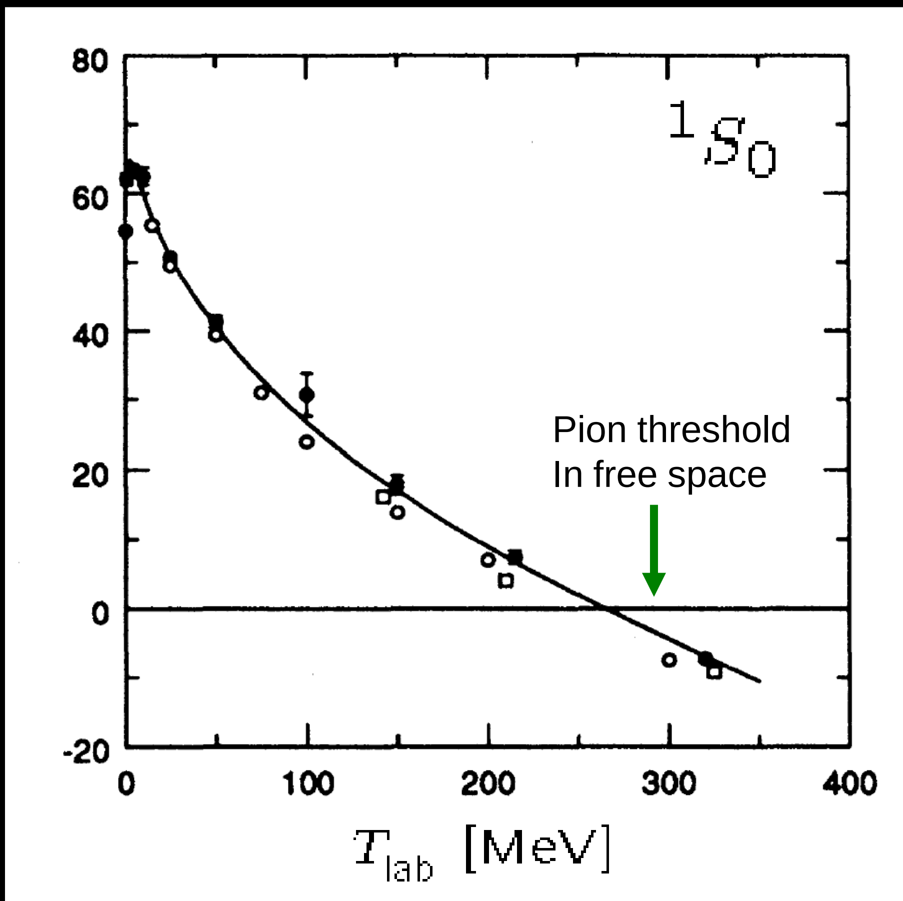
- YN force in quenched QCD:
  Nemura, Ishii, Aoki & T.H., Phys. Lett. B673 (2009) 136 [arXiv:0806.1094 [nucl-th]].

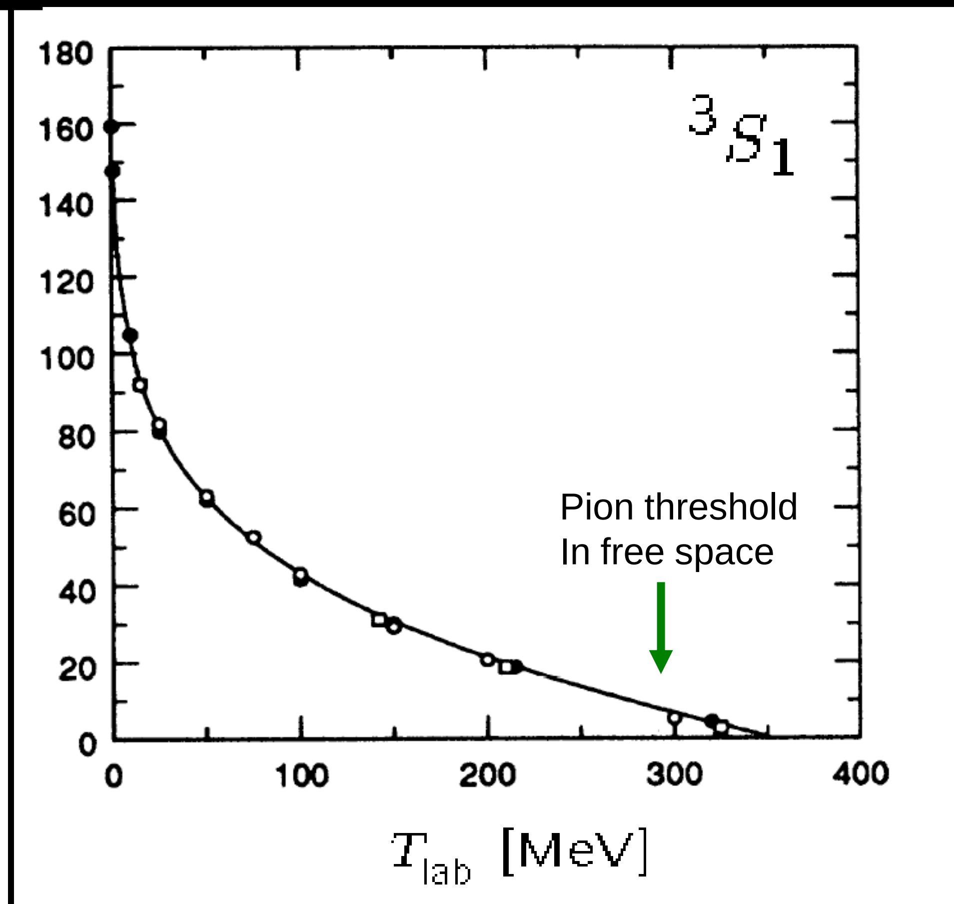
- NN force in full QCD:
  Ishii, Aoki & T.H. (for PACS-CS Coll.), arXiv: 0903.5497 [hep-lat]

- YN force in full QCD:
  Nemura, Ishii, Aoki & T.H. (for PACS-CS Coll.), arXiv: 0902.12251 [hep-lat]

Backup slides

NN phase shifts





Stoks et al., Phys.Rev. C48 (1993) 792

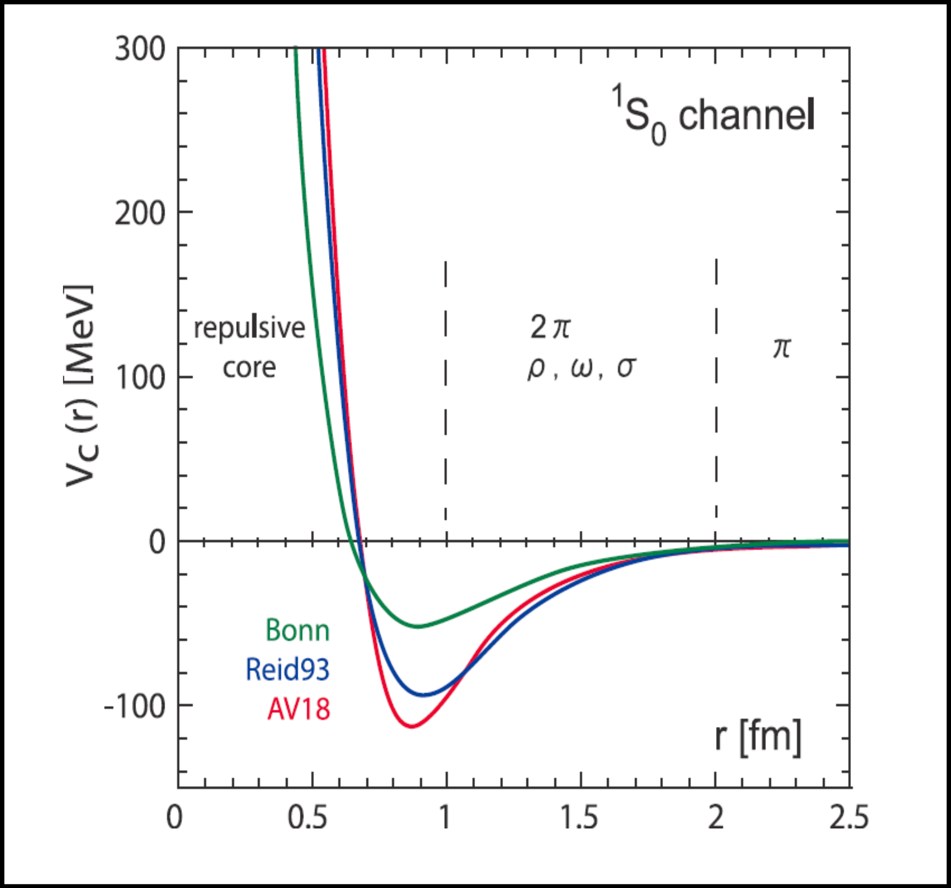
Phenomenological nuclear force below pion threshold

NN phase shifts → NN potential

$$V(r) = V_{\mathrm{C}}(r) + S_{12}\ V_{\mathrm{T}}(r) + \mathbf{L}\cdot\mathbf{S}\ V_{\mathrm{LS}}(r) + O(\nabla^2) + \cdots$$

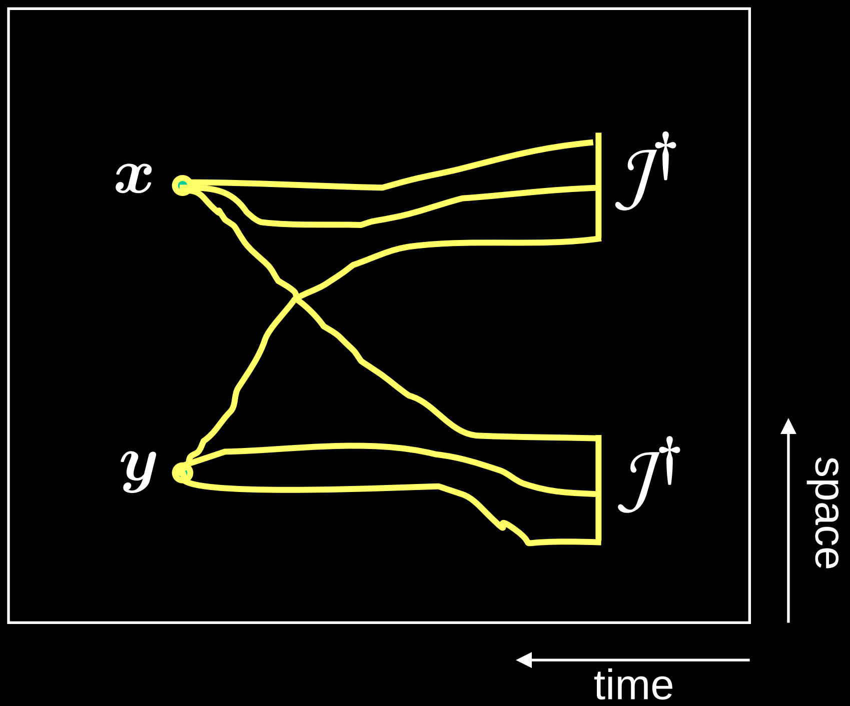
Okubo & Marshak (1958)

- Intermediate attraction
  - → nuclear binding
- Short range repulsion
  - → nuclear stability
- Strong tensor force
  - → deuteron binding
- Strong LS force
  - → p-wave neutron superfluidity
- 3-body forces
  - → nuclear binding/stability max. mass of N-stars



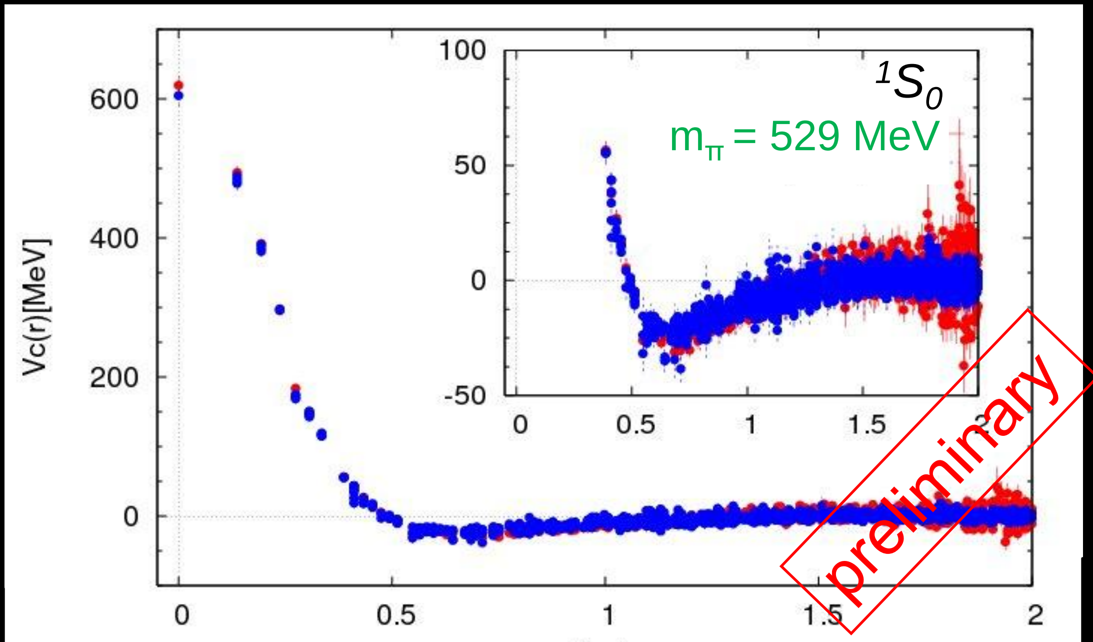
Measurement of ϕ (**r**) (s-wave)

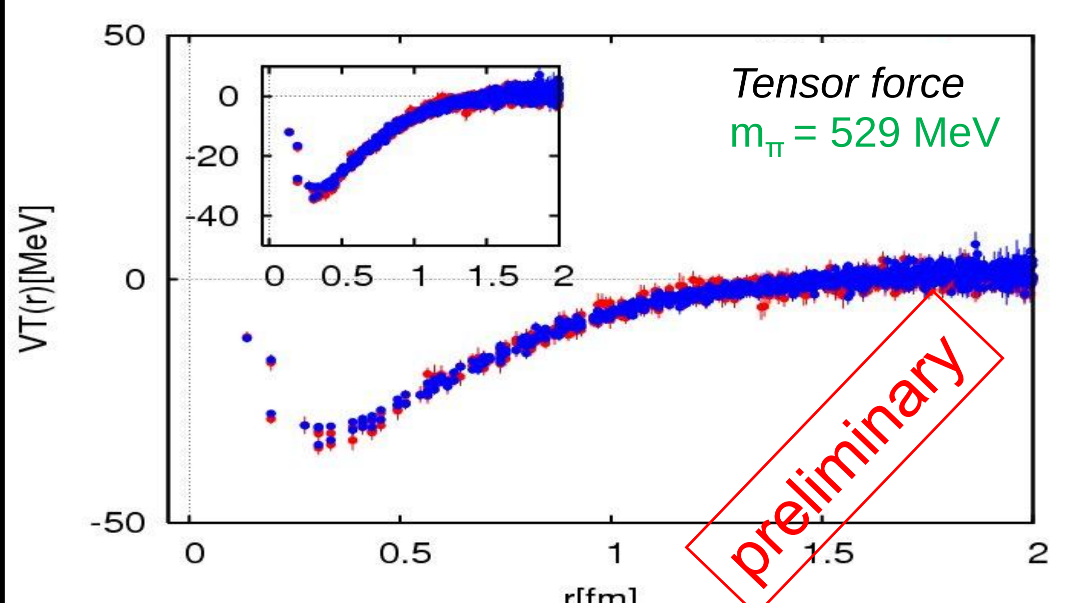
$$
\begin{aligned}
C_4(\mathbf{r};t) &= \langle N_1(\mathbf{x},t)N_2(\mathbf{y},t)\mathcal{J}_1^\dagger(0)\mathcal{J}_2^\dagger(0)\rangle \\
&= \sum_n \langle 0|N_1(\mathbf{x})N_2(\mathbf{y})|n\rangle A_n\, e^{-E_n t} \longrightarrow \phi(\mathbf{r})A_0 e^{-E_0 t}
\end{aligned}
$$

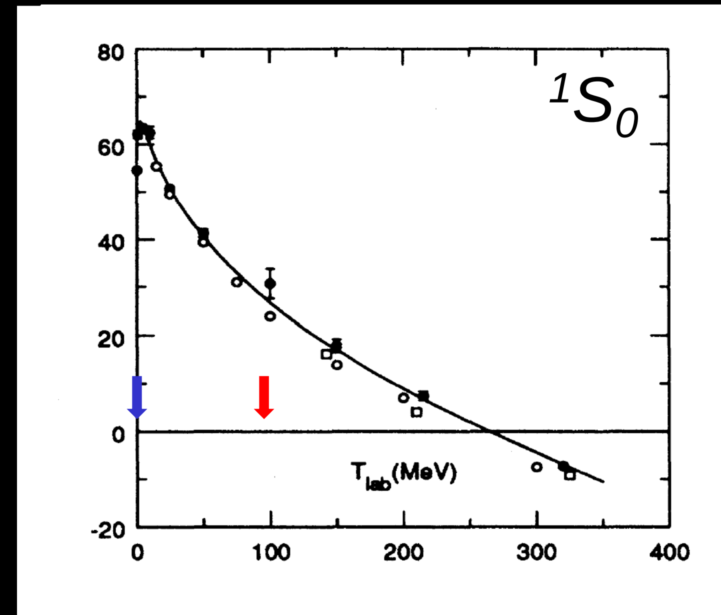


+ all possible combinations

velocity-dependece of $V_{C,T}(r)$







Next Generation National Supercomputing Facility
20 Pflops @ Kobe
(2011 partial operation, 2012 full operation)

http://www.nsc.riken.jp/index_j.html

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