

NN potential from Lorentz-invariant χ EFT

Renato Higa

Helmholtz-Institut für Strahlen- und Kernphysik (Theorie), and
Bethe Center for Theoretical Physics
University of Bonn



Chiral Dynamics 2009

NN potential from Lorentz-invariant χ EFT

Outline

- Motivation: one-baryon χ PT
- structure of the R-TPE
- comparison with the HB approach: expressions
- comparison with the HB approach: observables
 - ★ peripheral waves
 - ★ renormalization of the NN interaction:
deuteron properties and lower waves
- Summary

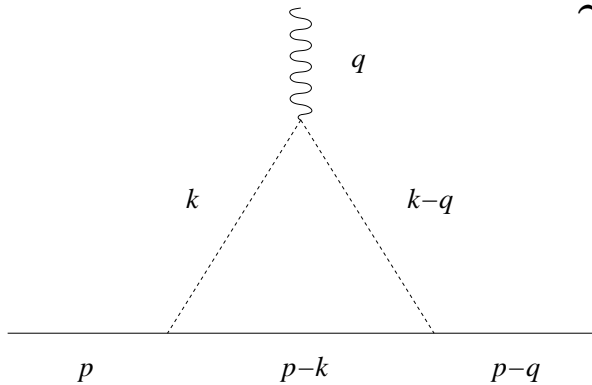


(one) baryon χ PT

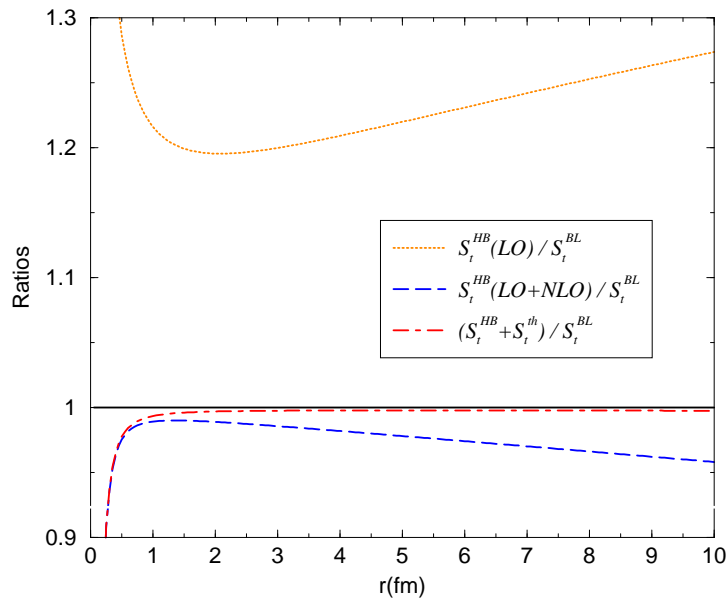
- Gasser, Sainio e Švarc: loops with dimensional regularization
→ violation of power counting: $\Delta\mathcal{L}_N = \mathcal{L}_N^{(2)} + \mathcal{L}_N^{(3)} + \Delta\mathcal{L}_N^{(0)} + \Delta\mathcal{L}_N^{(1)}$
- Jenkins and Manohar: heavy baryon expansion (HB- χ PT)
recovers power counting
- Ellis and Tang, Becher and Leutwyler: relativistic formalism
(see also Fuchs *et al.*, Schindler *et al.*, ...)
 - ★ I (low-energy contributions)
 - ★ R (high momenta contributions)
- R is analytic in all low-energy region → Taylor expansion (local operators)
- I exhibit a power counting rule
- q/m_N expansion of I → HB- χ PT



- triangle integral: q/m_N expansion fails near $t = 4m_\pi^2$



$$\begin{aligned} \gamma(t) &= \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{dt'}{(t'-t)} \frac{1}{16\pi m_N \sqrt{t'}} \arctan \frac{2m_N \sqrt{t'-4m_\pi^2}}{t-2m_\pi^2} \\ &\approx \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{dt'}{(t'-t)} \frac{1}{16\pi m_N \sqrt{t'}} \left\{ \left[\frac{\pi}{2} - \frac{(t'-2m_\pi^2)}{2m_N \sqrt{t'-4m_\pi^2}} \right]_{HB} \right. \\ &\quad \left. + \left[\frac{m_\pi \sqrt{t'}}{2m_N \sqrt{t'-4m_\pi^2}} - \frac{\sqrt{t'}}{2m_\pi} \arctan \frac{m_\pi^2}{m_N \sqrt{t'-4m_\pi^2}} \right]_{th} \right\}. \end{aligned}$$



◇ TPEP in coordinate space

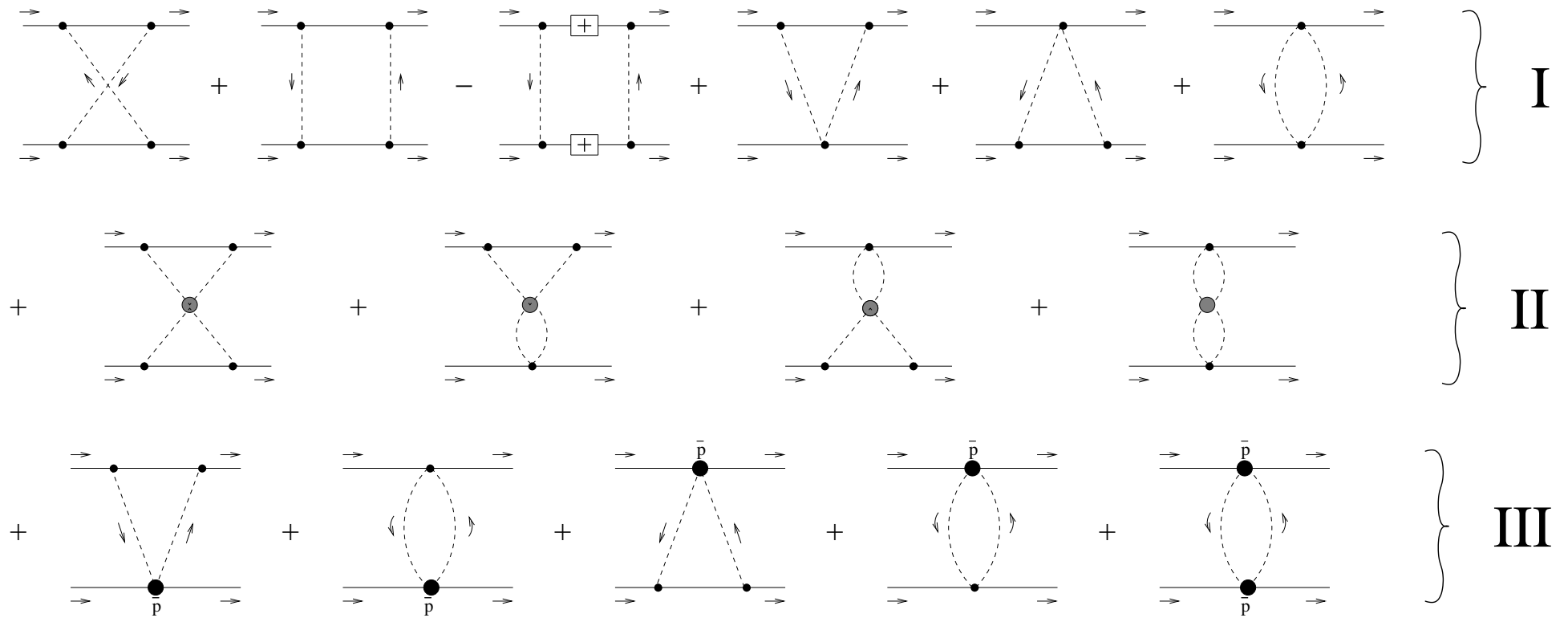
(RH, AC da Rocha, MR Robilotta, Phys.Rev.C 69, 034009, 2004)

convergence of the HB series
breaks down at large distances!



LI-TPE

RH and MR Robilotta, Phys.Rev. C 68, 024004 (2003)



Comparison with HB expressions

(Kaiser *et al.*, Kaiser, Entem and Machleidt, Epelbaum *et al.*)

- strategy: perform a **formal** $1/m_N$ expansion in our loop integrals
(despite the fact that it doesn't converge!)
- relatively small number of differences
- origins RH, [nucl-th/0411046](#)
 - ★ **dynamical equations** (our choice: Blankenbecler and Sugar)
 - ★ **two loop calculations** (not completely understood)



$$\begin{aligned}
V_C = V_C^+ &= \frac{3g_A^2}{16\pi f_\pi^4} \left\{ -\frac{g_A^2 \mu^5}{16m(4\mu^2 + \mathbf{q}^2)} + [2\mu^2(2c_1 - c_3) - \mathbf{q}^2 c_3] (2\mu^2 + \mathbf{q}^2) A(q) + \frac{g_A^2(2\mu^2 + \mathbf{q}^2) A(q)}{16m} [-3\mathbf{q}^2 + (4\mu^2 + \mathbf{q}^2)^\dagger] \right\} \\
&+ \frac{g_A^2 L(q)}{32\pi^2 f_\pi^4 m} \left\{ \frac{24\mu^6}{4\mu^2 + \mathbf{q}^2} (2c_1 + c_3) + 6\mu^4(c_2 - 2c_3) + 4\mu^2 \mathbf{q}^2(6c_1 + c_2 - 3c_3) + \mathbf{q}^4(c_2 - 6c_3) \right\} \\
&- \frac{3L(q)}{16\pi^2 f_\pi^4} \left\{ [-4\mu^2 c_1 + c_3(2\mu^2 + \mathbf{q}^2) + c_2(4\mu^2 + \mathbf{q}^2)/6]^2 + \frac{1}{45} (c_2)^2 (4\mu^2 + \mathbf{q}^2)^2 \right\} \\
&+ \frac{g_A^4}{32\pi^2 f_\pi^4 m^2} \left\{ L(q) \left[\frac{2\mu^8}{(4\mu^2 + \mathbf{q}^2)^2} + \frac{8\mu^6}{(4\mu^2 + \mathbf{q}^2)} - 2\mu^4 - \mathbf{q}^4 \right] + \frac{\mu^6/2}{(4\mu^2 + \mathbf{q}^2)} \right\} \\
&- \frac{3g_A^4 [A(q)]^2}{1024\pi^2 f_\pi^6} (\mu^2 + 2\mathbf{q}^2) (2\mu^2 + \mathbf{q}^2)^2 - \frac{3g_A^4(2\mu^2 + \mathbf{q}^2) A(q)}{1024\pi^2 f_\pi^6} \{4\mu g_A^2 (2\mu^2 + \mathbf{q}^2) + 2\mu (\mu^2 + 2\mathbf{q}^2)\} ,
\end{aligned}$$

$$\begin{aligned}
V_T = -\frac{3 V_T^+}{m^2} &= \frac{3g_A^4 L(q)}{64\pi^2 f_\pi^4} - \frac{g_A^4 A(q)}{512\pi f_\pi^4 m} [9(2\mu^2 + \mathbf{q}^2) + 3(4\mu^2 + \mathbf{q}^2)^\dagger] \\
&- \frac{g_A^4 L(q)}{32\pi^2 f_\pi^4 m^2} \left[\mathbf{z}^2/4 + 5\mathbf{q}^2/8 + \frac{\mu^4}{4\mu^2 + \mathbf{q}^2} \right] + \frac{g_A^2 (4\mu^2 + \mathbf{q}^2) L(q)}{32\pi^2 f_\pi^4} \left[(\tilde{d}_{14} - \tilde{d}_{15}) - (g_A^4/32 \pi^2 f_\pi^2)^* \right] ,
\end{aligned}$$

$$V_{LS} = -\frac{V_{LS}^+}{m^2} = -\frac{3g_A^4 A(q)}{32\pi f_\pi^4 m} [(2\mu^2 + \mathbf{q}^2) + (\mu^2 + 3\mathbf{q}^2/8)^\dagger] - \frac{g_A^4 L(q)}{4\pi^2 f_\pi^4 m^2} \left[\frac{\mu^4}{4\mu^2 + \mathbf{q}^2} + \frac{11}{32} \mathbf{q}^2 \right] - \frac{g_A^2 c_2 L(q)}{8\pi^2 f_\pi^4 m} (4\mu^2 + \mathbf{q}^2) ,$$

$$V_{\sigma L} = \frac{4 V_Q^+}{m^4} = -\frac{g_A^4 L(q)}{32\pi^2 f_\pi^4 m^2} ,$$



$$\begin{aligned}
W_C = V_C^- = & \frac{L(q)}{384\pi^2 f_\pi^4} \left[4\mu^2 (5g_A^4 - 4g_A^2 - 1) + \mathbf{q}^2 (23g_A^4 - 10g_A^2 - 1) + \frac{48g_A^4 \mu^4}{4\mu^2 + \mathbf{q}^2} \right] \\
& - \frac{g_A^2}{128\pi f_\pi^4 m} \left\{ \frac{3g_A^2 \mu^5}{4\mu^2 + \mathbf{q}^2} + A(q) (2\mu^2 + \mathbf{q}^2) \left[g_A^2 (4\mu^2 + 3\mathbf{q}^2) - 2(2\mu^2 + \mathbf{q}^2) + g_A^2 (4\mu^2 + \mathbf{q}^2)^\dagger \right] \right\} + \frac{\mathbf{q}^2 c_4 L(q)}{192\pi^2 f_\pi^4 m} \left[g_A^2 (8\mu^2 + 5\mathbf{q}^2) + (4\mu^2 + \mathbf{q}^2) \right] \\
& + \frac{16g_A^4 \mu^6}{768\pi^2 f_\pi^4 m^2} \frac{1}{4\mu^2 + \mathbf{q}^2} - \frac{L(q)}{768\pi^2 f_\pi^4 m^2} \left\{ (4\mu^2 + \mathbf{q}^2) z^2 + g_A^2 \left[\frac{48\mu^6}{4\mu^2 + \mathbf{q}^2} - 24\mu^4 - 12(2\mu^2 + \mathbf{q}^2) \mathbf{q}^2 + (16\mu^2 + 10\mathbf{q}^2) z^2 \right] \right. \\
& \left. + g_A^4 \left[z^2 \left(\frac{16\mu^4}{4\mu^2 + \mathbf{q}^2} - 7\mathbf{q}^2 - 20\mu^2 \right) - \frac{64\mu^8}{(4\mu^2 + \mathbf{q}^2)^2} - \frac{48\mu^6}{4\mu^2 + \mathbf{q}^2} + \frac{16\mu^4 \mathbf{q}^2}{4\mu^2 + \mathbf{q}^2} + 20\mathbf{q}^4 + 24\mu^2 \mathbf{q}^2 + 24\mu^4 \right] \right\} \\
& - \frac{L(q)}{18432\pi^4 f_\pi^6} \left\{ \left[192\pi^2 f_\pi^2 \tilde{d}_3 - \frac{(15+7g_A^4)^*}{5} \right] (4\mu^2 + \mathbf{q}^2) \left[2g_A^2 (2\mu^2 + \mathbf{q}^2) - 3/5(g_A^2 - 1)(4\mu^2 + \mathbf{q}^2) \right] \right. \\
& \left. + \left[6g_A^2 (2\mu^2 + \mathbf{q}^2) - (g_A^2 - 1) (4\mu^2 + \mathbf{q}^2) \right] \left[384\pi^2 f_\pi^2 \left((2\mu^2 + \mathbf{q}^2) (\tilde{d}_1 + \tilde{d}_2) + 4\mu^2 \tilde{d}_5 \right) + L(q) (4\mu^2 (1 + 2g_A^2) + \mathbf{q}^2 (1 + 5g_A^2)) \right. \right. \\
& \left. \left. - \left(\frac{\mathbf{q}^2}{3} (5 + 13g_A^2) + 8\mu^2 (1 + 2g_A^2) \right) + \left(2g_A^4 (2\mu^2 + \mathbf{q}^2) + \frac{2}{3} \mathbf{q}^2 (1 + 2g_A^2) \right)^* \right] \right\},
\end{aligned}$$

$$\begin{aligned}
W_T = -\frac{3}{m^2} V_T^- = & \frac{g_A^2 A(q)}{32\pi f_\pi^4} \left\{ \left(c_4 + \frac{1}{4m} \right) (4\mu^2 + \mathbf{q}^2) - \frac{g_A^2}{8m} \left[10\mu^2 + 3\mathbf{q}^2 - (4\mu^2 + \mathbf{q}^2)^\dagger \right] \right\} - \frac{c_4^2 L(q)}{96\pi^2 f_\pi^4} (4\mu^2 + \mathbf{q}^2) \\
& + \frac{c_4 L(q)}{192\pi^2 f_\pi^4 m} \left[g_A^2 (16\mu^2 + 7\mathbf{q}^2) - (4\mu^2 + \mathbf{q}^2) \right] - \frac{L(q)}{1536\pi^2 f_\pi^4 m^2} \left[g_A^4 \left(28\mu^2 + 17\mathbf{q}^2 + \frac{16\mu^4}{4\mu^2 + \mathbf{q}^2} \right) - g_A^2 (32\mu^2 + 14\mathbf{q}^2) + (4\mu^2 + \mathbf{q}^2) \right] \\
& - \frac{[A(q)]^2 g_A^4 (4\mu^2 + \mathbf{q}^2)^2}{2048\pi^2 f_\pi^6} - \frac{A(q) g_A^4 (4\mu^2 + \mathbf{q}^2)}{1024\pi^2 f_\pi^6} \mu (1 + 2g_A^2),
\end{aligned}$$

$$\begin{aligned}
W_{LS} = -\frac{1}{m^2} V_{LS}^- = & \frac{A(q)}{32\pi f_\pi^4 m} \left[g_A^2 (g_A^2 - 1) (4\mu^2 + \mathbf{q}^2) + g_A^4 (2\mu^2 + 3\mathbf{q}^2/4)^\dagger \right] + \frac{c_4 L(q)}{48\pi^2 m f_\pi^4} \left[g_A^2 (8\mu^2 + 5\mathbf{q}^2) + (4\mu^2 + \mathbf{q}^2) \right] \\
& + \frac{L(q)}{256\pi^2 m^2 f_\pi^4} \left[(4\mu^2 + \mathbf{q}^2) - 16g_A^2 (\mu^2 + 3\mathbf{q}^2/8) + \frac{4g_A^4}{3} \left(9\mu^2 + 11\mathbf{q}^2/4 - \frac{4\mu^4}{4\mu^2 + \mathbf{q}^2} \right) \right],
\end{aligned}$$

$$W_{\sigma L} \simeq 0,$$



Comparison with HB expressions

(Kaiser *et al.*, Kaiser, Entem and Machleidt, Epelbaum *et al.*)

- strategy: perform a formal $1/m_N$ expansion in our loop integrals (despite the fact that it doesn't converge!)
- relatively small number of differences
- origins RH, nucl-th/0411046
 - ★ dynamical equations (our choice: Blankenbecler and Sugar) (6)
 - ★ two loop calculations (not completely understood) (3)
- to isolate the $1/m_N$ expansion effect: ignore the above differences
- cutoff: $[1 - \exp(-cr^2)]^4 \times \text{TPEP}$

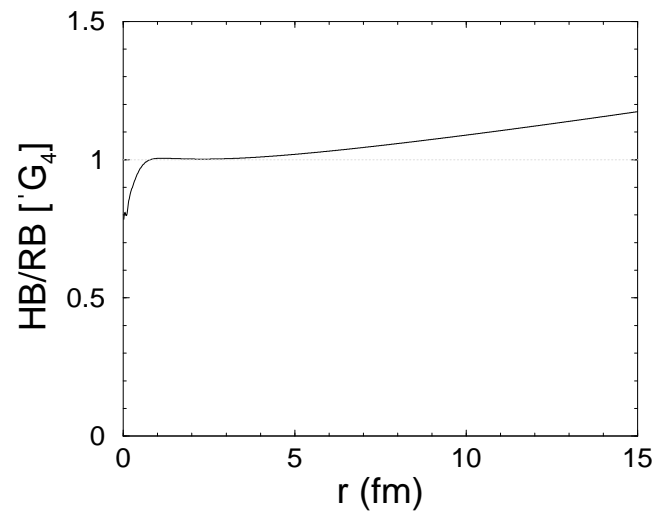
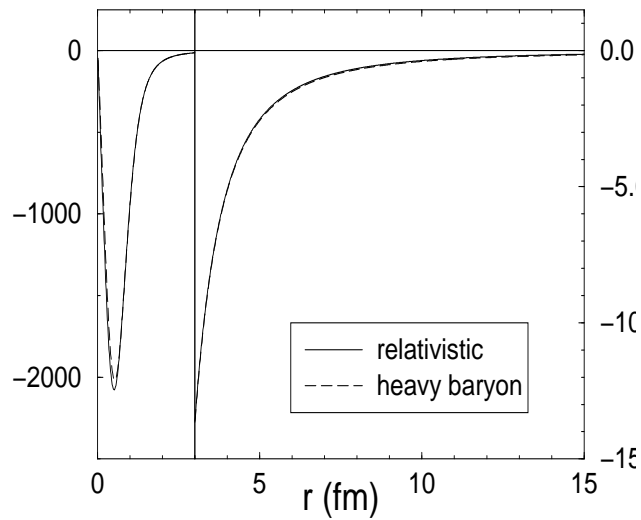
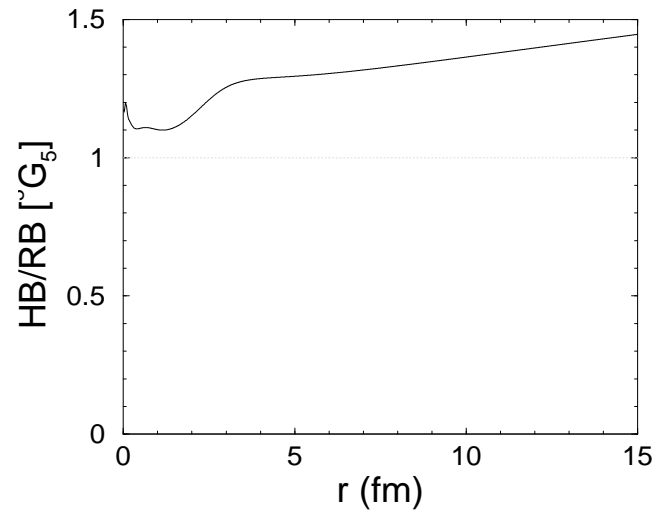
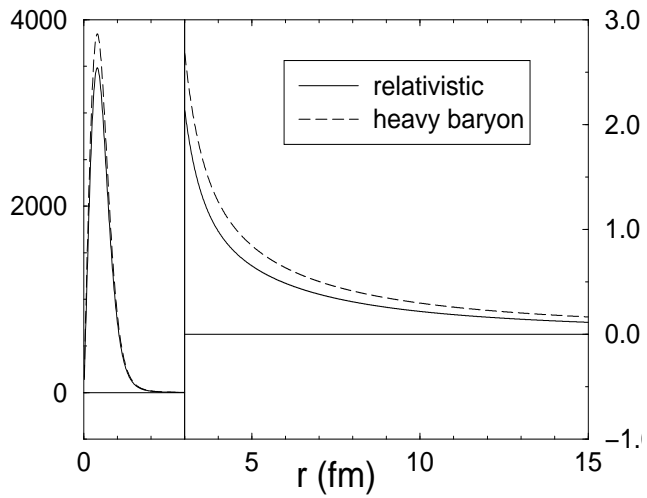


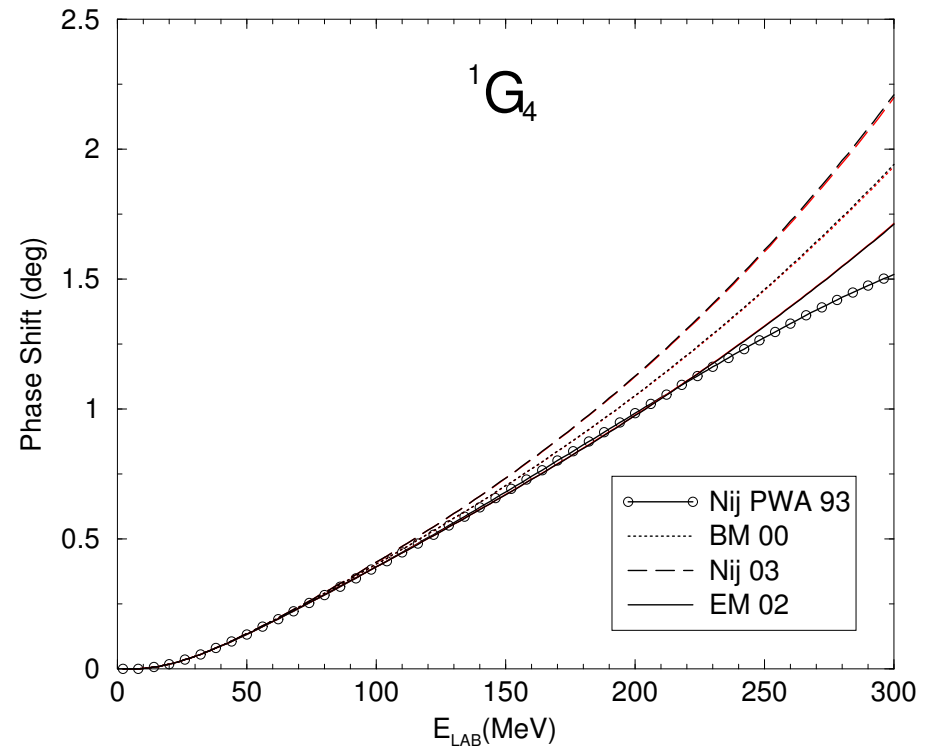
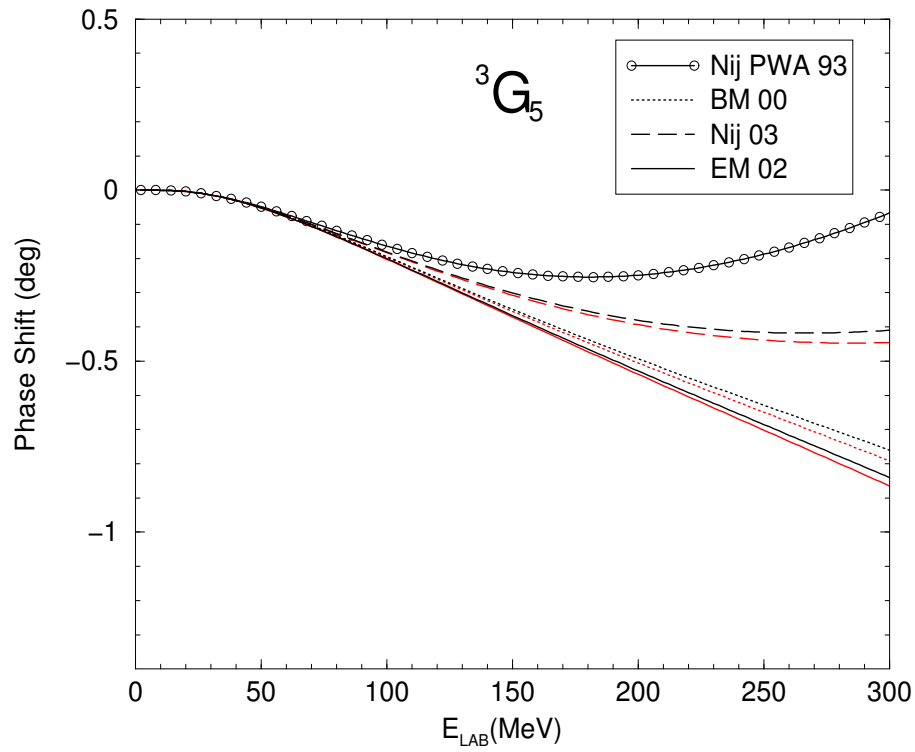
LECs

LEC	BM 00	Nij 03	EM 02
c_1	-0.81	-0.76	-0.81
c_2	8.43	3.20	3.28
c_3	-4.70	-4.78	-3.40
c_4	3.40	3.96	3.40
$\bar{d}_1 + \bar{d}_2$	3.06	3.06	3.06
\bar{d}_3	-3.27	-3.27	-3.27
\bar{d}_5	0.45	0.45	0.45
$\bar{d}_{14} - \bar{d}_{15}$	-5.65	-5.65	-5.65

d_i 's from Fettes *et al.*, Nucl.Phys. A 640, 199 (1998)







Comparison with HB expressions

(Kaiser *et al.*, Kaiser, Entem and Machleidt, Epelbaum *et al.*)

- strategy: perform a formal $1/m_N$ expansion in our loop integrals (despite the fact that it doesn't converge!)
- relatively small number of differences
- origins RH, nucl-th/0411046
 - ★ dynamical equations (our choice: Blankenbecler and Sugar) (6)
 - ★ two loop calculations (not completely understood) (3)
- to isolate the $1/m_N$ expansion effect: ignore the above differences
- renormalization of the singular OPE+TPE



Singular potentials with boundary conditions

- look at the divergence of the potential as $r \rightarrow 0$
 - ★ repulsive \rightarrow prediction
 - ★ attractive \rightarrow requires physical input
- multi-channels: behavior of the eigen-potentials
 - ★ orthogonality of the wave functions near the origin

(see M. Pavón Valderrama's talk on Tuesday)

Set	Source	$c_1(\text{GeV}^{-1})$	$c_3(\text{GeV}^{-1})$	$c_4(\text{GeV}^{-1})$
Set IV	EM 03	-0.81	-3.20	5.40
Set η	RH, Valderrama, Arriola	-0.81	-3.80	4.50



Wave	α NijmII (Reid93)	OPE	HB-NLO	HB-NNLO Set I, II & III	HB-NNLO SetIV	RB-TPE
1S_0	-23.727(-23.735)	Input	Input	Input	Input	Input
3P_0	-2.468(-2.469)	Input	—	Input	—	—(*)
1P_1	2.797(2.736)	—	—	Input	—	—
3P_1	1.529(1.530)	—	Input	Input	Input	Input
3S_1	5.418(5.422)	Input	—	Input	Input	Input
3D_1	6.505(6.453)	—	—	Input	Input	—
E_1	1.647(1.645)	—	—	Input	Input	—
1D_2	-1.389(-1.377)	—	Input	Input	Input	Input
3D_2	-7.405(-7.411)	Input	Input	Input	Input	Input
3P_2	-0.2844(-0.2892)	Input	Input	Input	Input	Input
3F_2	-0.9763(-0.9698)	—	—	Input	—	—(*)
E_2	1.609(1.600)	—	—	Input	—	—(*)
1F_3	8.383(8.365)	—	—	Input	—	Input
3F_3	2.703(2.686)	—	Input	Input	Input	Input
3D_3	-0.1449(-0.1770)	Input	—	Input	Input	Input
3G_3	4.880(4.874)	—	—	Input	Input	—
E_3	-9.695(-9.683)	—	—	Input	Input	—
1G_4	-3.229(-3.210)	—	Input	Input	Input	Input
3G_4	-19.17(-19.14)	Input	Input	Input	Input	Input
3F_4	-0.01045(-0.01053)	Input	Input	Input	Input	Input
3H_4	-1.250(-1.240)	—	—	Input	—	—(*)
E_4	3.609(3.586)	—	—	Input	—	—(*)
1H_5	28.61(28.57)	—	—	Input	—	Input
3H_5	6.128(6.082)	—	Input	Input	Input	Input
3G_5	-0.0090(-0.010)	Input	—	Input	Input	Input
3I_5	10.68(10.66)	—	—	Input	Input	—
E_5	-31.34(-31.29)	—	—	Input	Input	—

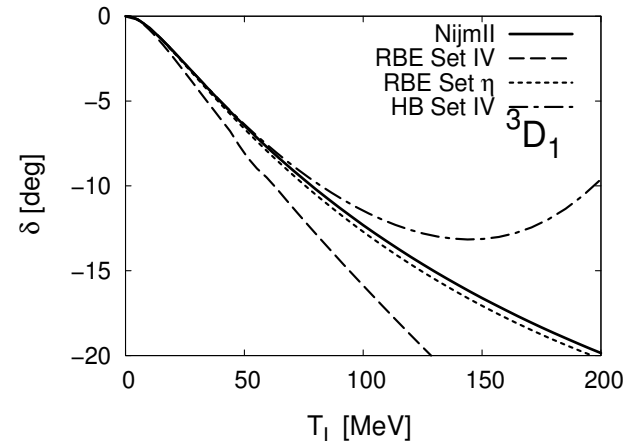
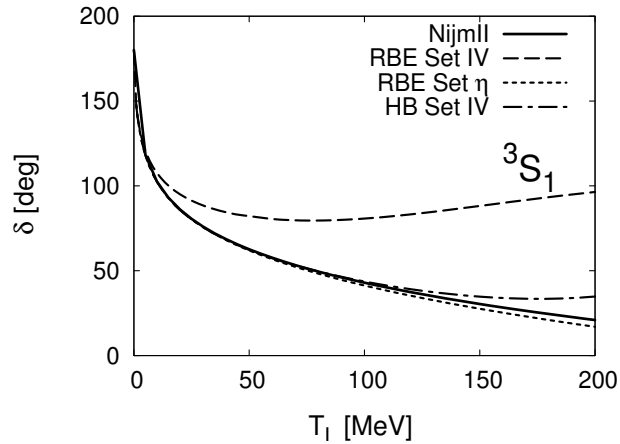
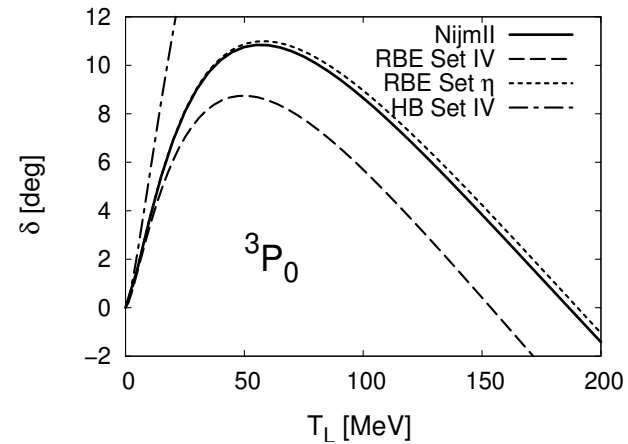
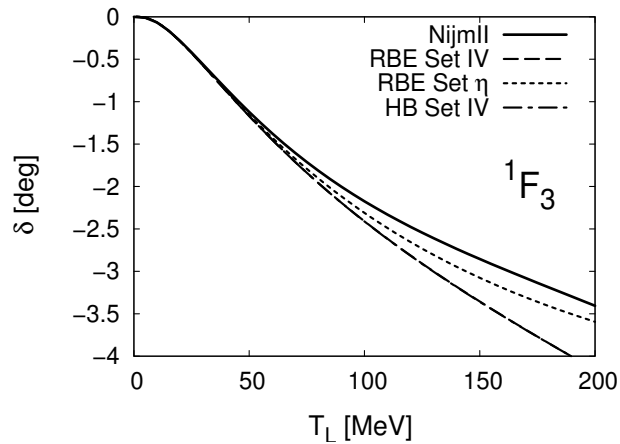


Singular potentials with boundary conditions

Set	$\gamma(\text{fm}^{-1})$	η	$A_S(\text{fm}^{-1/2})$	$r_d(\text{fm})$	$Q_d(\text{fm}^2)$	P_D
OPE	Input	0.02634	0.8681(1)	1.9351(5)	0.2762(1)	7.88(1)%
HB Set IV	Input	Input	0.884(4)	1.967(6)	0.276(3)	8(1)%
RBE Set IV	Input	0.03198(3)	0.8226(5)	1.8526(10)	0.3087(2)	22.99(13) %
RBE Set η	Input	0.02566(1)	0.88426(2)	1.96776(1)	0.2749(1)	5.59(1) %
NijmII	0.231605	0.02521	0.8845(8)	1.9675	0.2707	5.635%
Reid93	0.231605	0.02514	0.8845(8)	1.9686	0.2703	5.699%
Exp.	0.231605	0.0256(4)	0.8846(9)	1.971(6)	0.2859(3)	5.67(4)%



Singular potentials with boundary conditions



Summary

- derivation of R-TPE to $O(q^4)$ (NN-N³LO) based on BL studies
- **problem with the HB formalism:** shows up in NN at **large distances**
- $1/m_N$ expansion of **R-TPEP** reproduce **most** of the HB results
(**discrepancy: 2-loop results**)
- peripheral waves: R vs. HB is **small**, dependence on the πN LECs is **large**
- renormalization of the singular R-TPE: good description of observables with **less counterterms** (\sim half) than HB-TPE
- improvement of codes, determination of short-distance parameters



- C.A. da Rocha and M.R. Robilotta, Phys. Rev. C 49, 1818 (1994), Phys. Rev. C 52, 531 (1995), Nucl. Phys. A 615, 391 (1997),
- R.H. and M.R. Robilotta, Phys. Rev. C 68, 024004 (2003),
- R.H., C.A. da Rocha, M.R. Robilotta, Phys. Rev. C 69, 034009 (2004), arXiv: nucl-th/0501076
- R.H., arXiv: nucl-th/0411046.
- R.H., M. Pavón Valderrama, and E. Ruiz Arriola, Phys. Rev. C 77, 034003 (2008).

