A method to measure the $\bar{K}N$ scattering length in lattice QCD

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Motivation

The $\bar{K}N$ scattering length

- Contradictory experimental information on $a_{\bar{K}N}$ from scattering data and kaonic hydrogen level shifts
- Conflicting data on kaonic hydrogen from DEAR and KEK collaboration
- Model dependence in the description of $\bar{K}N$ scattering (e.g. by unitarized coupled-channel ChPT)
- Information about $\bar{K}N$ scattering is needed in the theoretical description of the interaction of kaons with nuclear matter

$\Rightarrow$ Extraction of $a_{\bar{K}N}$ from lattice QCD will be highly welcome !

Standard way of extracting the scattering length in lattice QCD

Lüscher Formula:

- Measure the ground state energy level $E = E(L)$ of the two-particle system in a finite box of size L
- For large L

$$E \sim \frac{a}{L^3}\{1 + c_1 \frac{a}{L} + c_2 \frac{a^2}{L^2}\} + O(L^{-6})$$

where c_1 and c_2 are known real numbers

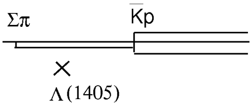
$\curvearrowright$ Extraction of scattering length a

$\Rightarrow$ Works only for elastic case with real scattering length

Extraction of parameters for the inelastic case

$\bar{K}N$ scattering with $I = 0$:

- Two channels: $\bar{K}N = 1$, $\Sigma\pi = 2$
- $\Lambda(1405)$ between the two thresholds



Two-channel Lippmann-Schwinger eqn. in infinite volume:

$$\begin{aligned} T_{11} &= H_{11} + H_{11} i q_1 T_{11} + H_{12} i q_2 T_{21} \\ T_{21} &= H_{21} + H_{21} i q_1 T_{11} + H_{22} i q_2 T_{21} \end{aligned}$$

- H_{ij} determine the position of the $\Lambda(1405)$
- scattering length

$$a_{\bar{K}N} \doteq a_{11} = H_{11}(s) + \frac{i q_2(s)(H_{12}(s))^2}{1 - i q_2(s) H_{22}(s)}\Bigg|_{s=(m_N+M_K)^2}$$

The same system in a finite volume

$$\int \frac{d^3k}{(2\pi)^3}(\ldots) \to \frac{1}{L^3}\sum_k (\ldots) \qquad \vec{k} = \frac{2\pi\vec{n}}{L} \quad \vec{n} \in \mathbb{Z}^3$$

Lüscher Zeta-function:

$$: \qquad iq \to \frac{2}{\sqrt{\pi}L} Z_{00}(1;k^2) \quad k = \frac{Lq}{2\pi}$$

$$Z_{00}(1;k^2) = \frac{1}{\sqrt{4\pi}} \lim_{s\to 1} \sum_{\vec{n}\in\mathbb{Z}^3} \frac{1}{(\vec{n}^2 - k^2)^s}$$

Lippmann-Schwinger eqn. in a finite volume:

$$\begin{aligned} T_{11} &= H_{11} + H_{11} Z_{00}(1;k_1^2) T_{11} + H_{12} Z_{00}(1;k_2^2) T_{21} \\ T_{21} &= H_{21} + H_{21} Z_{00}(1;k_1^2) T_{11} + H_{22} Z_{00}(1;k_2^2) T_{21} \end{aligned}$$

Spectrum in a finite volume

The spectrum is found from the poles of the T-Matrix:

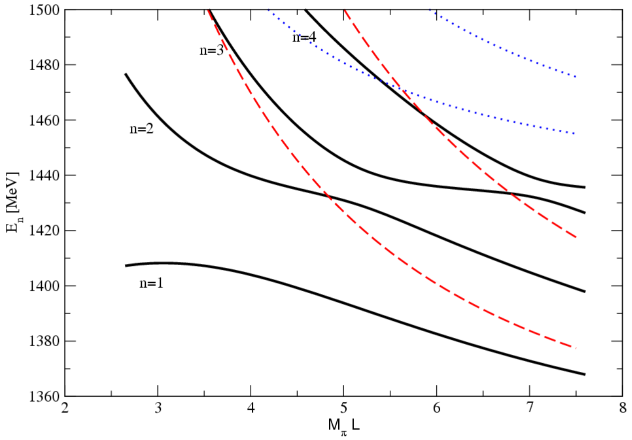
$$\mathscr{M}_{ij} T_{jk} = H_{ik} \qquad (i,j,k = 1,2)$$

$$\det \mathscr{M}(H_{11}, H_{12}, H_{22}; L; E) = 0, \quad E^2 = s$$

$$\Rightarrow E_n = E_n(H_{11}, H_{12}, H_{22}; L)$$

Now H_{11}, H_{12}, H_{22} can be determined from a fit to $E_n = E_n(L)$ from lattice calculations

Spectrum in a finite volume



- H_{11}, H_{12}, H_{22} from the ground-state and the first excited level $\curvearrowright a_{\bar{K}N}$
- Parameters of $\Lambda(1405)$ can also be extracted (the pole in the infinite volume)

Conclusions

- Generalisation of the Lüscher Formula to the multi-channel case:
  - Permits extraction of complex scattering lengths from lattice data
  - Allows to determine parameters of resonances in the vicinity of multiple thresholds, here the $\Lambda(1405)$