Two-Flavor ChPT for Hyperons

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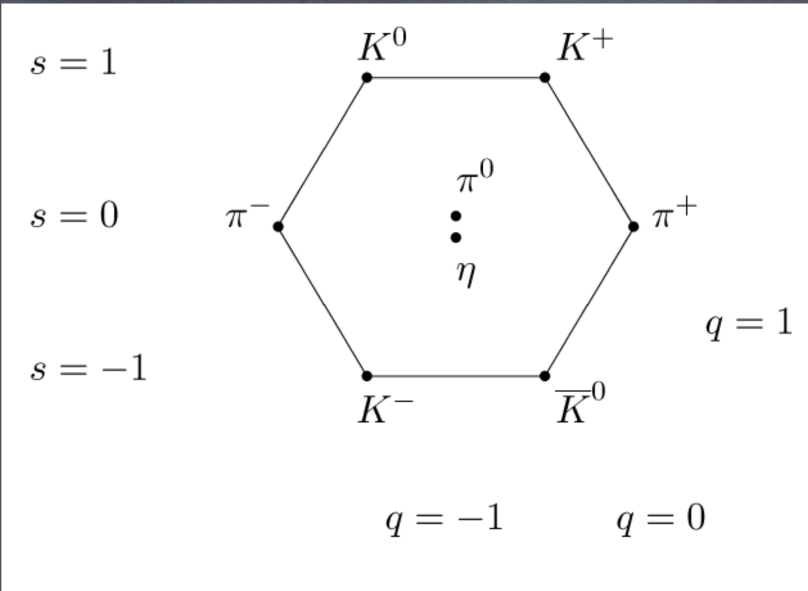
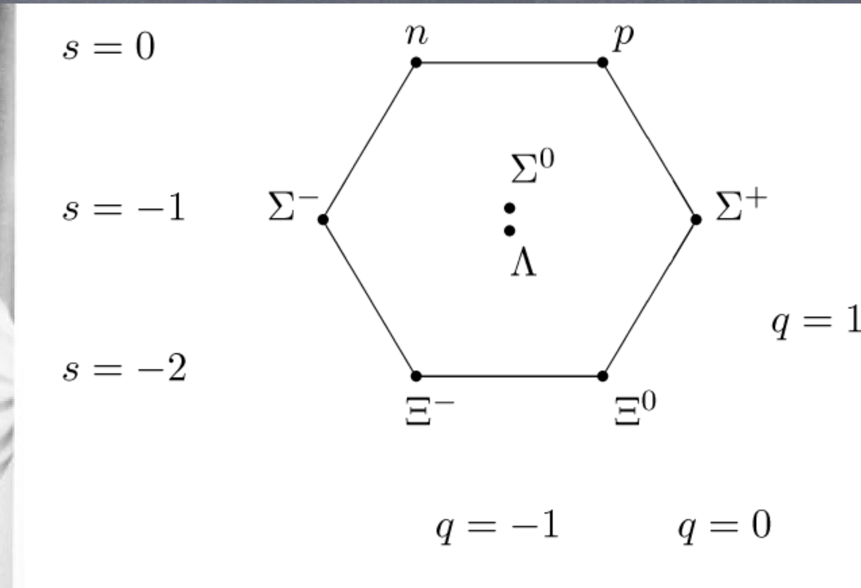
Outline

- Why SU(3)? / Why not SU(3)?
  - Gell-Mann Okubo relations
  - Baryon masses
  - Baryon axial couplings
- SU(2) theories for hyperons
- Prospects
  - Insight for lattice QCD data
  - Assess convergence of ChPT
  - SU(3) relations

Why SU(3)?

Low-lying states seem to organize themselves into SU(3) multiplets





Octet of pseudoscalar mesons

Octet of spin-1/2 baryons

Light mesons suggest: spontaneous chiral symmetry breaking

$$m_u,\, m_d,\, m_s \ll \Lambda_{QCD} \qquad SU(3)_L \times SU(3)_R \to SU(3)_V$$

Gell-Mann Okubo Relation:

LO ChPT

$$\Delta_{GMO}(m_\phi^2) = \frac{4}{3}m_K^2 - m_\eta^2 - \frac{1}{3}m_\pi^2 = 0$$

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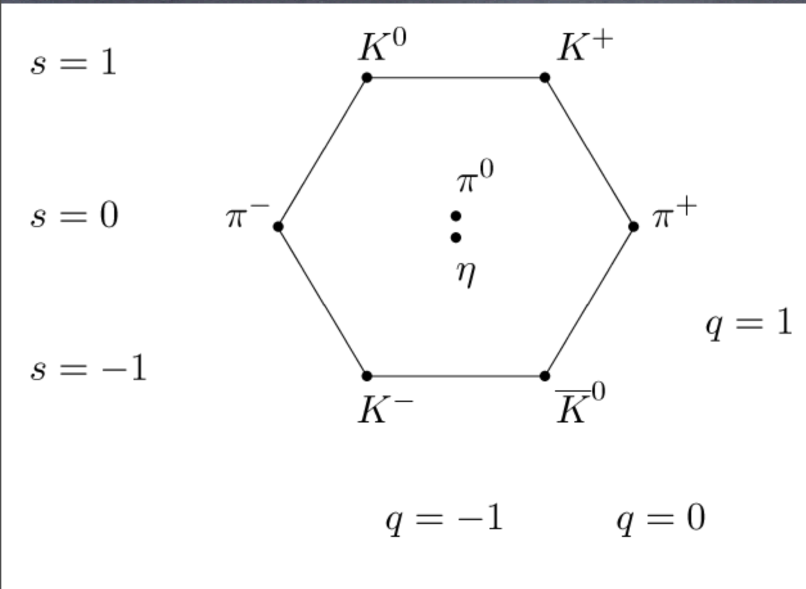

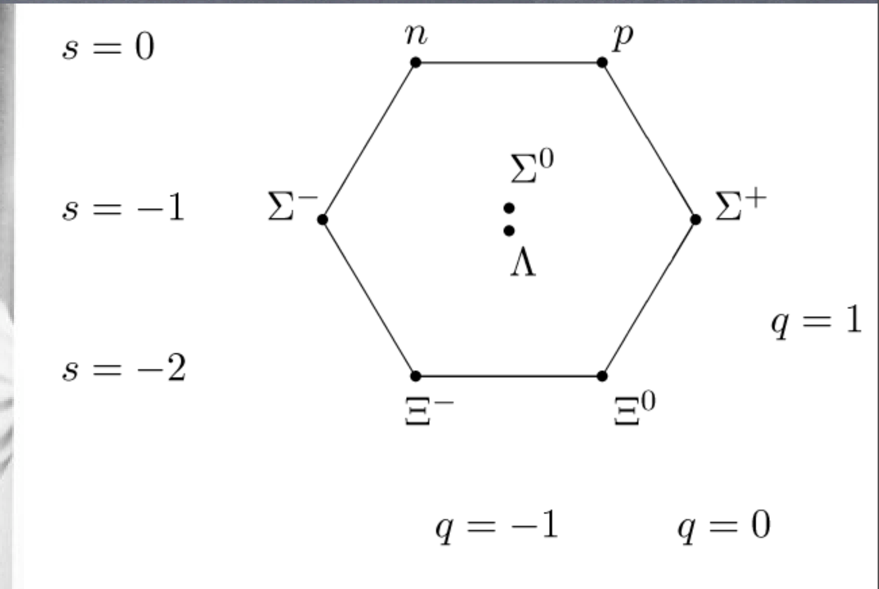

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Gell-Mann Okubo Relation:

LO ChPT

$$\Delta_{GMO} / \sum n_\phi m_\phi^2 = 14.5\%$$

Exp.

NLO ChPT

Gell-Mann Okubo Relation: Baryon Masses

$$< \overline{B}[m_q, B] >, \; < \overline{B}, \{m_q, B\} >, \; < \overline{B} B >< m_q >$$

Expanding about SU(3) chiral limit -> parameter free relation

LO HBChPT

$$M_{GMO} = M_\Lambda + \frac{1}{3} M_\Sigma - \frac{2}{3} M_N - \frac{2}{3} M_\Xi = 0$$

Gell-Mann Okubo Relation: Baryon Masses

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Expanding about SU(3) chiral limit -> parameter free relation

LO HBChPT

$$M_{GMO} / \sum_B n_B M_B = 0.90\%$$

Exp.

NLO HBChPT

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$$< \overline{B}[m_q, B] >, \;\; < \overline{B}, \{m_q, B\} >, \;\; < \overline{B} B >< m_q >$$

Expanding about SU(3) chiral limit -> parameter free relation

LO HBChPT

$$M_{GMO} / \sum_B n_B M_B = 0.90\%$$

Exp.

NLO HBChPT

$$M_{GMO} = \frac{4}{3\Lambda_\chi^2}\left[\pi(D^2 - 3F^2)\Delta_{GMO}(m_\phi^3) - \frac{1}{6}C^2\Delta_{GMO}\Big(\mathcal{F}(m_\phi, \Delta, \mu)\Big)\right]$$

μ -dependence is only superficial

Thy.

$$M_{GMO} / \sum_B n_B M_B = \begin{array}{ll} 0.79\% & \{\mathrm{ChPT}: \; D = 0.61, \; F = 0.40, \; C = 1.2\} \\ 1.12\% & \{\mathrm{LQCD}: \; D = 0.72, \; F = 0.45, \; C = 1.61\} \\ 1.29\% & \{SU(6): \; D = 3/4, \; F = 1/2, \; C = 3/2\} \end{array}$$

Why not SU(3)? Baryon Masses

Fix parameters $\{\mathrm{ChPT}: D = 0.61,\ F = 0.40,\ C = 1.2\}$

NLO contributions

$$\delta M_N(\mu = \Lambda_\chi)/M_N = -39\%$$

$$\delta M_\Lambda(\mu = \Lambda_\chi)/M_\Lambda = -67\%$$

$$\delta M_\Sigma(\mu = \Lambda_\chi)/M_\Sigma = -89\%$$

$$\delta M_\Xi(\mu = \Lambda_\chi)/M_\Xi = -98\%$$

Kaon, eta contributions large & increase with strangeness

$$m_s \sim \Lambda_{QCD}$$

SU(3) expansion precarious

& empirical evidence from LQCD

Why SU(2)?

Schematic SU(3) Expansion of Cascade Mass:

$$M_\Xi = M^{SU(3)} + a m_K^2 + b m_K^3 + \ldots$$

Large Kaon contributions

$$m_K^2 = \frac{1}{2} m_\pi^2 + \frac{1}{2} m_{\eta_s}^2 \qquad m_{\eta_s} = 672\,\texttt{MeV}$$

Reorganize! $\quad m_\pi / m_{\eta_s} = 0.2 \ll 1$

$$M_\Xi = M^{SU(3)} + a' m_{\eta_s}^2 + a'' m_\pi^2 + b' m_{\eta_s}^3 + b'' m_{\eta_s} m_\pi^2 + b''' m_\pi^3 \left(\frac{m_\pi}{m_{\eta_s}} \right) + \ldots$$

$$M_\Xi = M_\Xi^{SU(2)} \quad + \quad \alpha\, m_\pi^2 \quad + \quad \beta\, m_\pi^3 \quad + \quad \ldots$$

Expansion of Cascade Mass about the SU(2) chiral limit

$$m_u, m_d \ll m_s \sim \Lambda_{QCD}$$

Why SU(2)?

Expansion parameters: SU(3) HBChPT

$$\frac{p}{\Lambda_\chi} \sim \frac{m_\pi}{\Lambda_\chi} \sim \frac{\Delta}{\Lambda_\chi} \sim \varepsilon$$

$\frac{1}{2}$ →

$$\frac{m_K}{\Lambda_\chi} \sim \frac{m_\eta}{\Lambda_\chi} \sim \varepsilon$$

SU(2) HBChPT

$$\frac{p}{\Lambda_\chi} \sim \frac{m_\pi}{\Lambda_\chi} \sim \frac{\Delta_S}{\Lambda_\chi} \sim \varepsilon$$

Baryon multiplets: SU(3) HBChPT

$8 \; B$

$10 \; T$

SU(2) HBChPT

$S = 0 \quad 2\; N,\; 4\; \Delta$

$S = 1 \quad 1\; \Lambda,\; 3\; \Sigma,\; 3\; \Sigma^*$

$S = 2 \quad 2\; \Xi,\; 2\; \Xi^*$

$S = 3 \quad 1\; \Omega$

Axial Couplings: SU(3) HBChPT

$D,\; F,\; C,\; H$

SU(2) HBChPT

$S = 0, \quad g_A,\; g_{\Delta N},\; g_{\Delta\Delta}$

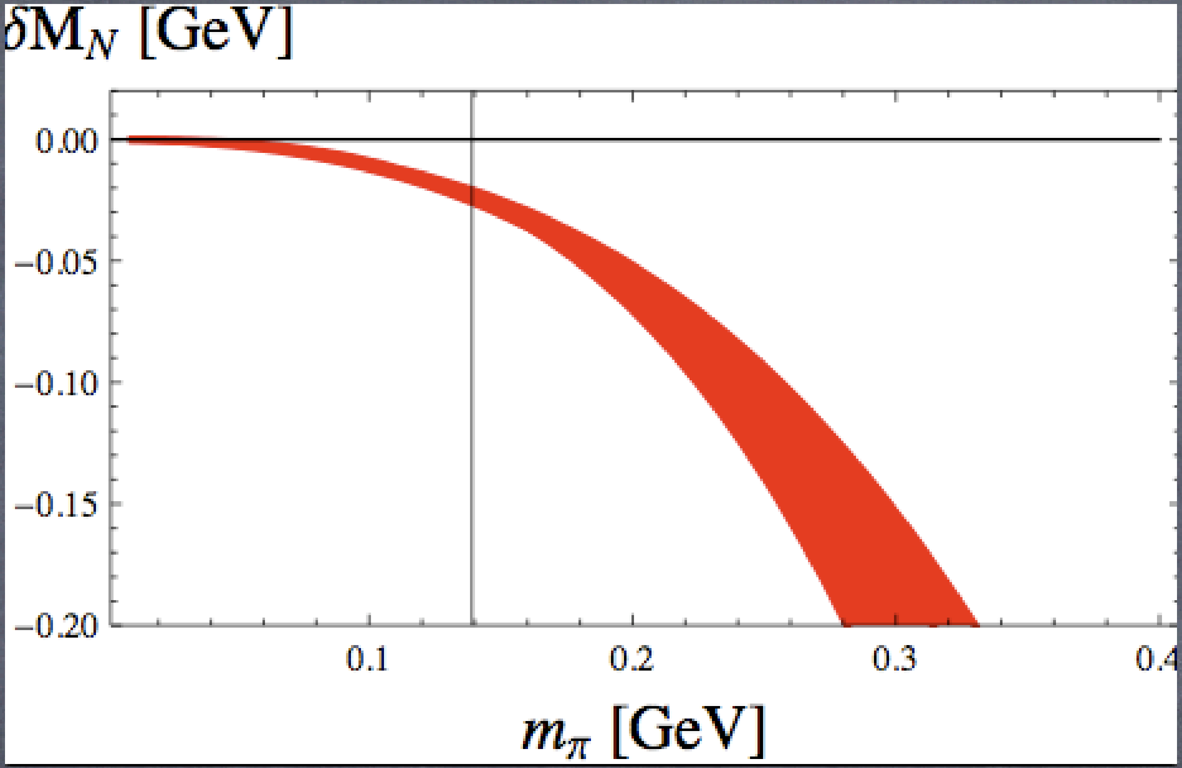
$S = 1, \quad g_{\Sigma\Sigma},\; g_{\Lambda\Sigma},\; g_{\Lambda\Sigma^*},$

$g_{\Sigma\Sigma^*},\; g_{\Sigma^*\Sigma^*}$

$S = 2, \quad g_{\Xi\Xi},\; g_{\Xi\Xi^*},\; g_{\Xi^*\Xi^*}$

Baryon Masses in SU(2) BCT, Walker-Loud PLB669 (2008)

NLO contributions



Marked improvement over SU(3): -39%

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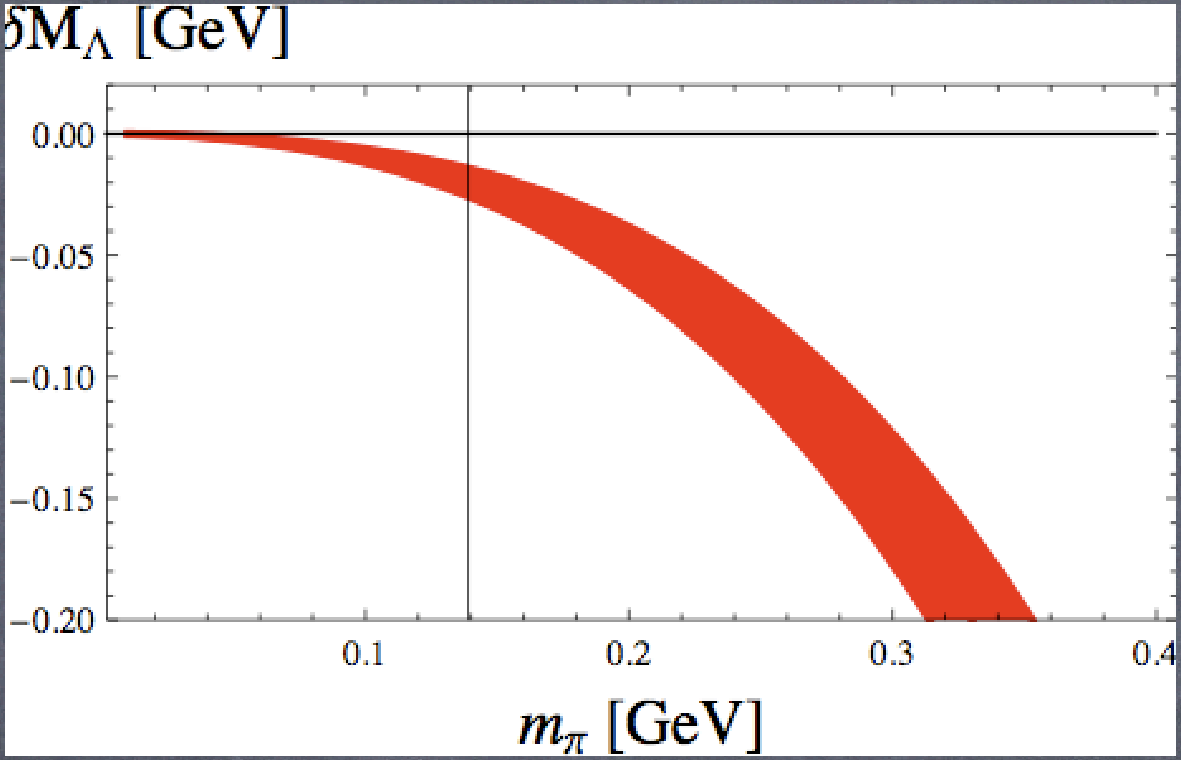
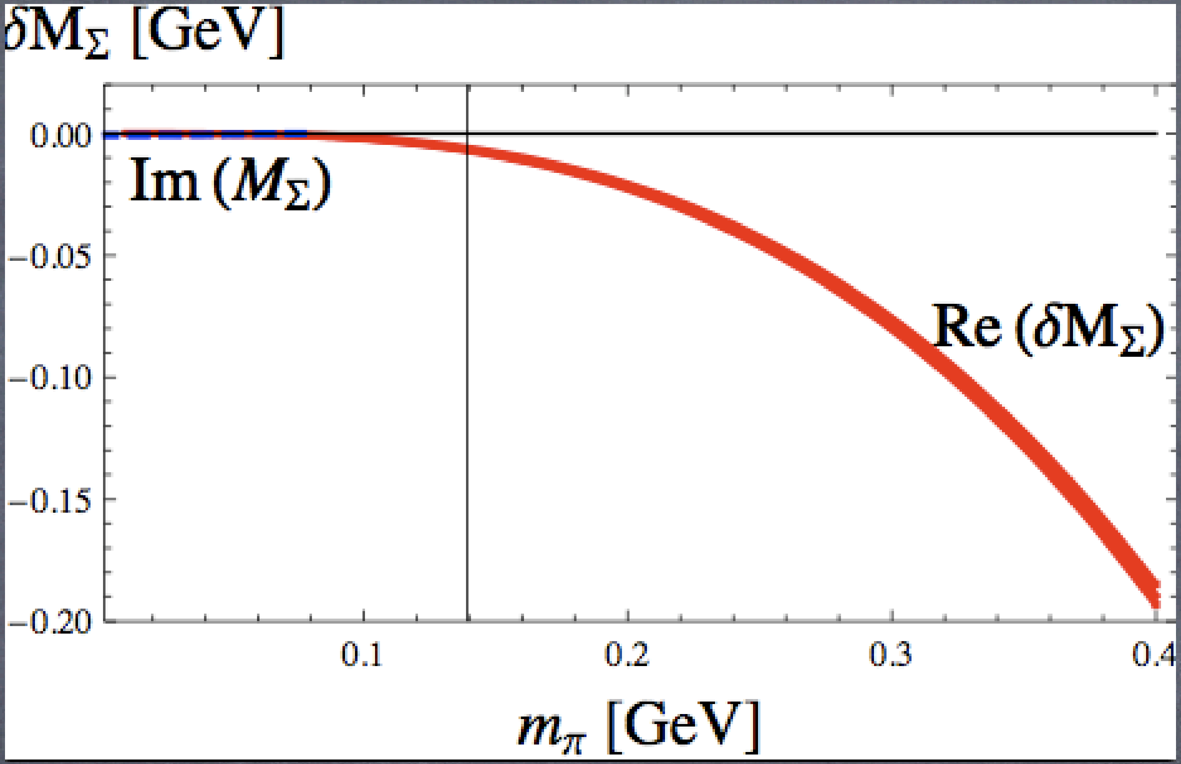


Marked improvement over SU(3): -67%

Baryon Masses in SU(2) BCT, Walker-Loud PLB669 (2008)

NLO contributions



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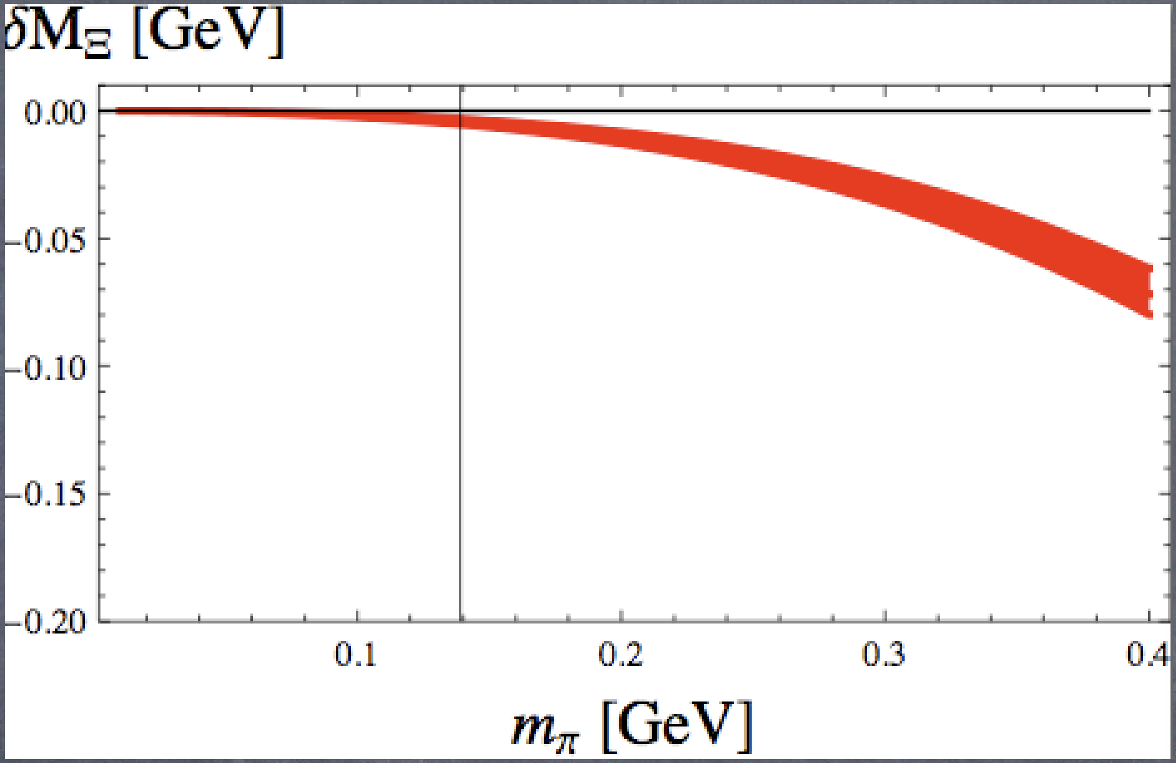
Baryon Masses in SU(2) BCT, Walker-Loud PLB669 (2008)

NLO contributions



Marked improvement over SU(3): -98%

Baryon Masses in SU(2) BCT, Walker-Loud PLB669 (2008)

NLO contributions

Marked improvement over SU(3)

Trend opposite SU(3): greater strangeness, better convergence

$$m_\pi / M_S$$

$$g_A = 1.25,\ g_{\Sigma\Sigma} = 0.78,\ g_{\Xi\Xi} = 0.24$$

$$g_{\Delta N} = 1.48,\ g_{\Sigma^*\Sigma} = 0.76,\ g_{\Xi^*\Xi} = 0.69$$

Gell-Mann Okubo Relations: Axial Charges

Jiang, BCT PRD77 (2008)

Expanding about SU(3) chiral limit -> parameter free relations

8 $\Delta I = 1 \, \mathrm{and} \, \Delta S = 1$ axial charges, but only 6 NLO operators

$$\Delta g = 2g_{NN} - g_{N\Lambda} - g_{N\Sigma} - g_{\Lambda\Sigma} - g_{\Sigma\Sigma} + 2g_{\Sigma\Xi}$$

$$\Delta G = 2g_{NN} + 2g_{\Xi\Xi} - 2g_{\Lambda\Sigma} + g_{N\Sigma} + g_{\Lambda\Xi} + g_{\Sigma\Xi} - g_{N\Lambda}$$

With D, F, C, and H as input:

$$\Delta g = -0.0035$$
$$\Delta G = -0.017$$

Validity of SU(3) ChPT?

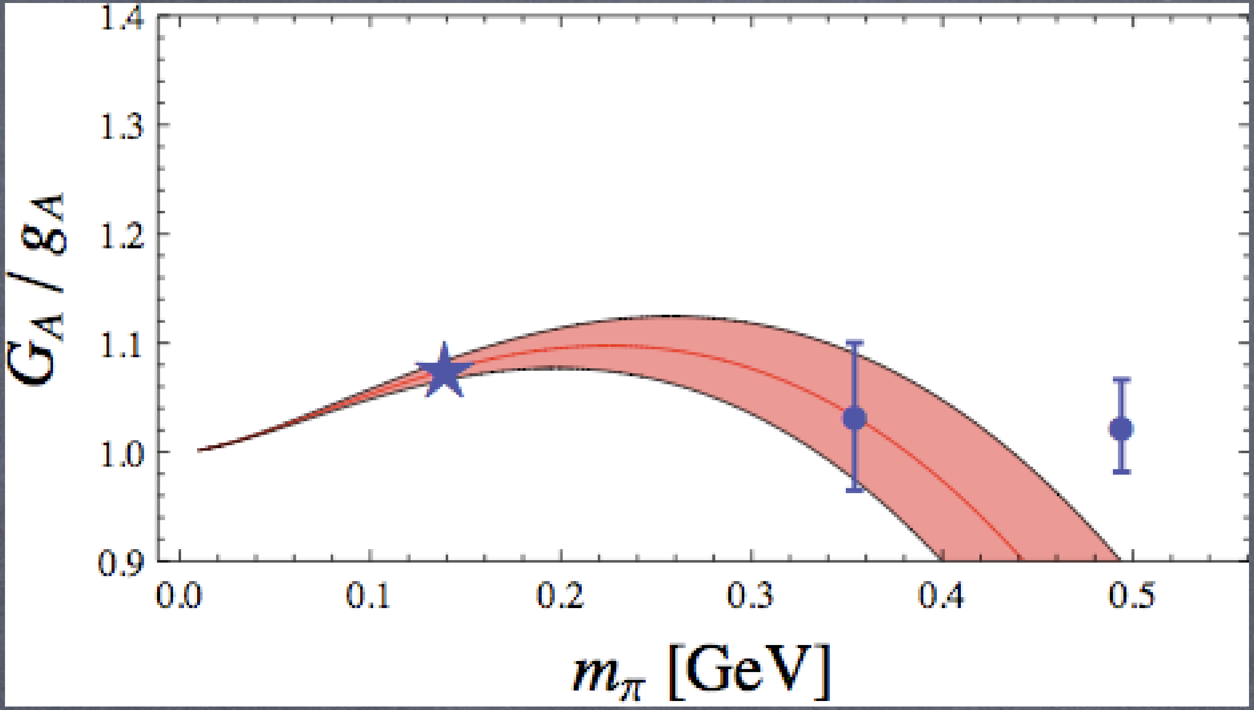
large strange meson loops contributing to individual axial charges

LQCD data does not support SU(3) extrapolation Lin, Orginos PRD78 (2009)

Perhaps SU(3) ChPT applies to $\Delta g, \; \Delta G$?

Axial Charges in SU(2) Jiang, BCT arXiv:0905.0857

NLO calculations, band from est. NNLO



LQCD data
Lin, Orginos PRD79 (2009)

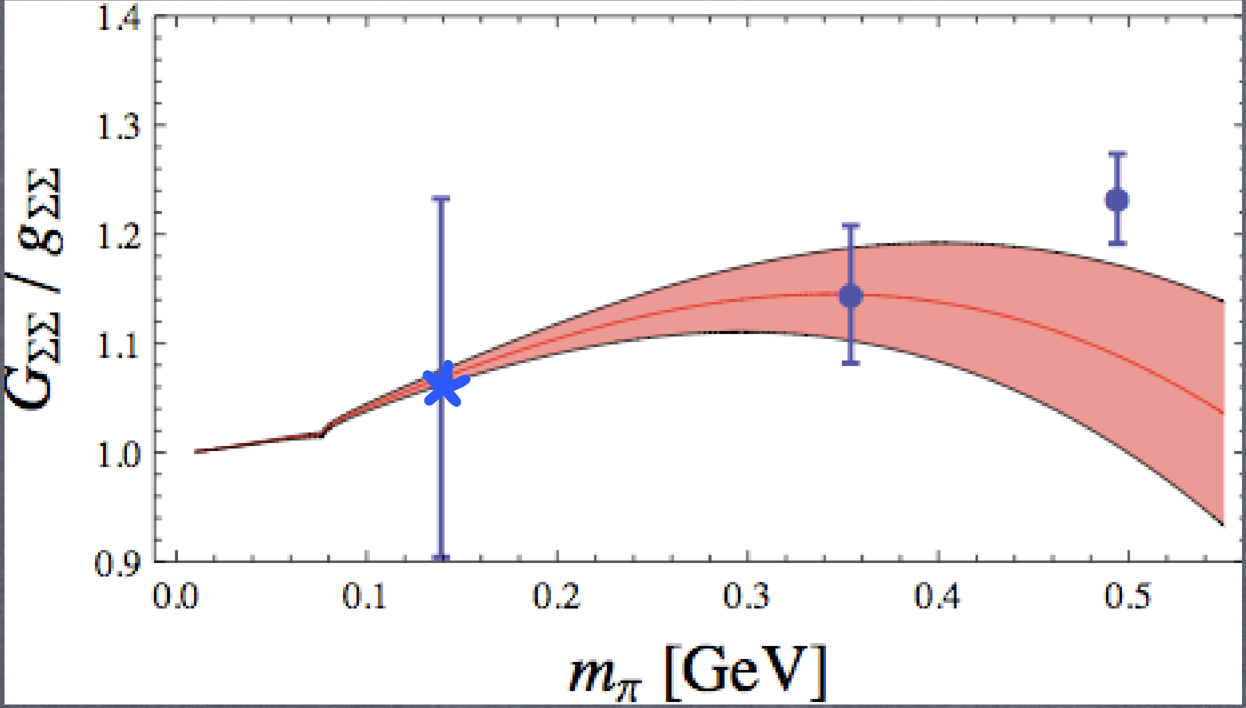
Marked improvement over behavior in SU(3)

Input: physical pt. & lightest LQCD pt., other couplings phen.

Local term: $A_{NN}(\mu)\dfrac{m_\pi^2}{\Lambda_\chi^2}$ $\qquad A_{NN}(\Lambda_\chi) = -12.0$

Axial Charges in SU(2) Jiang, BCT arXiv:0905.0857

NLO calculations, band from est. NNLO



LQCD data
Lin, Orginos PRD79 (2009)

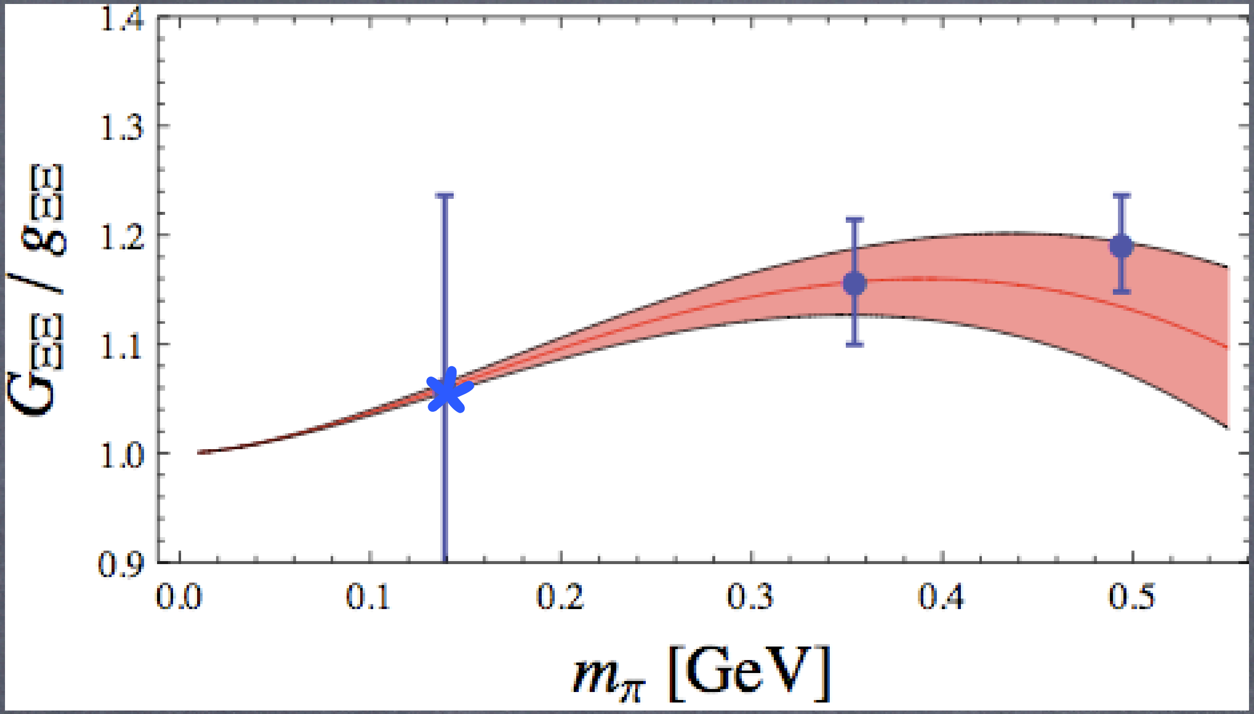
Marked improvement over behavior in SU(3)

Input: latt. extrap. & lightest LQCD pt., other couplings phen.

Local term: $A_{\Sigma\Sigma}(\mu)\dfrac{m_\pi^2}{\Lambda_\chi^2}$ $\qquad A_{\Sigma\Sigma}(\Lambda_\chi) = -2.9$

Axial Charges in SU(2) Jiang, BCT arXiv:0905.0857

NLO calculations, band from est. NNLO



LQCD data
Lin, Orginos PRD79 (2009)

Marked improvement over behavior in SU(3)

Input: latt. extrap. & lightest LQCD pt., other couplings phen.

Local term: $A_{\Xi\Xi}(\mu) \dfrac{m_\pi^2}{\Lambda_\chi^2}$ $\qquad$ $A_{\Xi\Xi}(\Lambda_\chi) = -0.22$

Axial Charges in SU(2) Jiang, BCT arXiv:0905.0857

NLO calculations, band from est. NNLO

Marked improvement over behavior in SU(3)

More natural results with increasing strangeness

Local terms:

$$A_{NN}(\Lambda_\chi) = -12.0$$
$$A_{\Sigma\Sigma}(\Lambda_\chi) = -2.9$$
$$A_{\Xi\Xi}(\Lambda_\chi) = -0.22$$

Summary

Baryon masses, axial couplings, . . .
receive sizable kaon, eta contributions in SU(3) ChPT

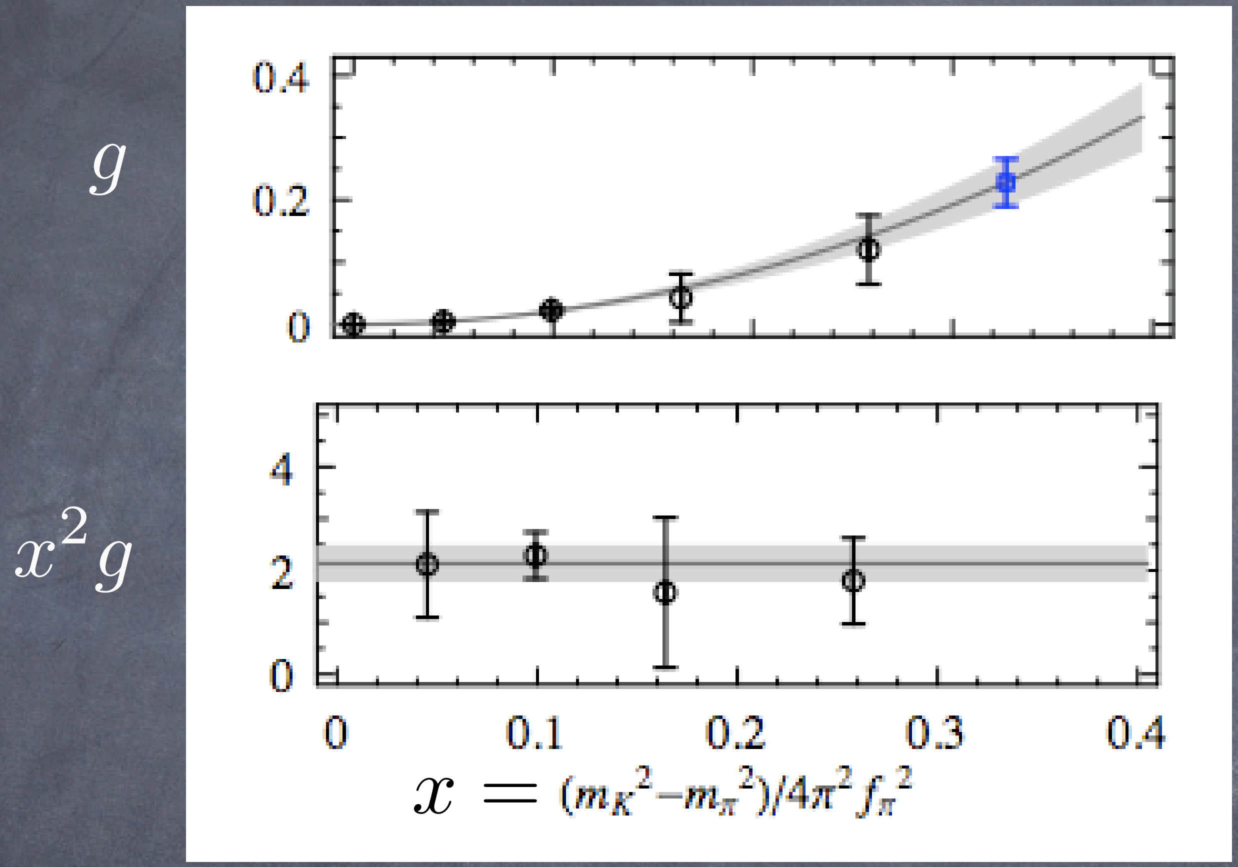
Convergence of SU(3) ChPT requires delicate balance
-large cancellations between terms of unnatural size

Baryon mass GMO relation is remarkable
SU(3) efficacious for quantities protected from large SU(3) breaking corrections?

Summary

- SU(2) ChPT for strange hadrons eliminates large loop contributions, behaves better with increasing strangeness
- SU(2) ChPT is ideal for extrapolation of lattice QCD data: strange quark mass is fixed near physical value, only pion mass extrapolation needed
- Challenges: demonstrate improved convergence using lattice data in chiral regime, determine LECs of SU(2), ultimately understand whether SU(3) is a tool for baryons

In defense of the lattice extrapolation



LQCD data

Lin, Orginos PRD79 (2009)

Conservative: lattice extrap. likely consistent with an SU(2) analysis

Cannot perform SU(2) analysis: next-to-lightest pion is 500 MeV