Relations between SU(2) and SU(3)-LECs in χPT at two-loop level

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Contents

Introduction

Chiral Perturbation Theory (χPT) is
Effective Field Theory of QCD at low energies

$$\mathcal{L}_{\rm QCD} \xrightarrow{E << M_\rho} \mathcal{L}_{\rm eff}$$

- **Exploits the chiral symmetry of QCD and its spontaneous breaking ;**
- **$\mathcal{L}_{\rm eff}$ is expressed in terms of the Goldstone bosons:**

  SU(2) $\Longrightarrow$ pions, SU(3) $\Longrightarrow$ pions, kaons, eta ;
- **Includes external sources v, a, p, s ;**
- **$\mathcal{L}_{\rm eff}$ is a series built according to the chiral power-counting ;**

Introduction

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots$$

$\mathcal{L}_2 \Longrightarrow$ Weinberg 1979

$\mathcal{L}_4 \Longrightarrow$ Gasser, Leutwyler 1984,1985

$\mathcal{L}_6 \Longrightarrow$ Fearing, Scherer 1996; Bijnens, Colangelo, Ecker 1999,2000

$$\mathcal{L}_2 = \frac{F^2}{4} < u_\mu u^\mu + \chi_+ >$$

u_μ contains Goldstone bosons coupled with external vector and axial fields

χ_+ contains Goldstone bosons coupled with external scalar and pseudoscalar fields

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$$\mathcal{L}_4^{SU_2} = \sum_{i=1}^{10} \ell_i K_i \qquad \mathcal{L}_6^{SU_2} = \sum_{i=1}^{57} c_i P_i$$

(57→ 56 arXiv:0705.0576 [hep-ph])

$$\mathcal{L}_4^{SU_3} = \sum_{i=1}^{12} L_i X_i \qquad \mathcal{L}_6^{SU_3} = \sum_{i=1}^{94} C_i Y_i$$

Introduction

- $\mathcal{L}_{\rm eff}$ contains a number of coupling constants, called **L**ow **E**nergy **C**onstant**s** **(LECs)** which are not fixed by chiral symmetry ;
- **LECs** are independent of the light quark masses $\mathbf{m_{u,d}}$. They describe the influence of heavy degrees of freedom ;
- Calculations with $\mathcal{L}_{\rm eff}$ give an expansion in quark masses and momenta:

Chiral perturbation theory $(\chi\mathrm{PT})$

Gasser, Leutwyler 1984,1985

- $\chi\mathrm{PT}_2$ gives an expansion around $\mathbf{m_u = m_d = 0}$ whereas $\chi\mathrm{PT}_3$ - around $\mathbf{m_u = m_d = m_s = 0}$;

Introduction

- **At small external momenta, these two expansions should be equivalent $\Longrightarrow$ one can express the LECs in $\chi\mathrm{PT}_2$ through the ones in $\chi\mathrm{PT}_3$ and the strange quark mass m_s ;**
- **Such relations give an additional information on the values of LECs ;**
- **The procedure of findings the relations between SU(2) and SU(3) LECs is called as "matching" ;**
- **The matching at one-loop level was done by**

  (Gasser, Leutwyler 1985)
- **The matching at two-loop level was done by**

  (Gasser, Haefeli, Ivanov, Schmid 2007,2009)

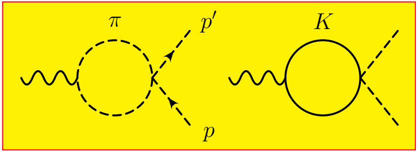
Example of matching at one-loop

$$\langle \pi^+(p')\,|\,\tfrac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)|\pi^+(p)\rangle = (p+p')_\mu F_V(t)\ ;\ t = (p'-p)^2\,,$$

In the chiral limit $m_u = m_d = 0$:

2 flavours : $$F_{V,2}(t) = 1 + \frac{t}{F^2}\Phi(t,0;d) - \frac{\ell_6 t}{F^2}$$

3 flavours : $$F_{V,3}(t) = 1 + \frac{t}{F_0^2}\left[\Phi(t,0;d) + \frac{1}{2}\Phi(t,M_K;d)\right] + \frac{2L_9 t}{F_0^2}$$



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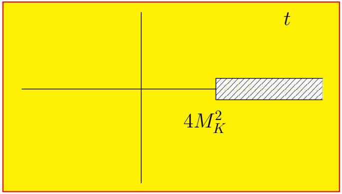

$$\Phi(t,M_K;d) = \sum_{n=0}^{\infty} \Phi_n(M_K,d)\left(\frac{t}{M_K^2}\right)^n$$

Example of matching at one-loop

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Drop terms of order "t" and higher in the expansion of $\Phi(t, M_K; d)$.
It is seen that $F_{V,3}(t)$ reduces to $F_{V,2}(t)$ if we put $F = F_0$ and

$$-\ell_6 = 2\,L_9 + \frac{1}{2}\Phi(0, M_K, d).$$

Example of matching at one-loop

$$\langle \pi^+(p') \,|\, \tfrac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)|\pi^+(p)\rangle = (p + p')_\mu F_V(t) \;;\; t = (p' - p)^2 \,,$$

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$$-\ell_6 = 2\,L_9 + \frac{1}{2}\Phi(0, M_K, d).$$

At $d = 4$, this equation gives the relation between renormalized LECs

$$\ell_6^r(\mu) = -2L_9^r(\mu) + \frac{1}{192\pi^2}(\ln B_0 m_s/\mu^2 + 1).$$

Gasser, Leutwyler (85)

Matching at two loops

- **One can get the matching for SU(2)-LECs at order p^2, p^4 from available two-loop calculations of the various matrix elements in $\chi\mathrm{PT}_3$**

- **But the matching for SU(2)-LECs at order p^6 in this manner requires a tremendous amount of two-loop calculations in $\chi\mathrm{PT}_{2,3}$.**

- **Therefore, we have developed a generic method based on the path integral formulation of $\chi\mathrm{PT}$.**

The idea of method

- **Reduce χPT_3 to χPT_2 by imposing two–flavor limit restrictions:**
  - * **the external sources of χPT_3 are restricted to the two–flavor subspace**
  - * **put $m_u = m_d = 0$, since the LECs of χPT are independent of $m_{u,d}$**
  - * **restrict by small external momenta $|p^2| << M_K^2$**
- **The generating functional of χPT_3 in the two–flavor limit equals now the one of χPT_2, i.e.**

$$Z^{SU(2)} = Z^{SU(3)}|_{SU(2)-\mathrm{limit}}$$

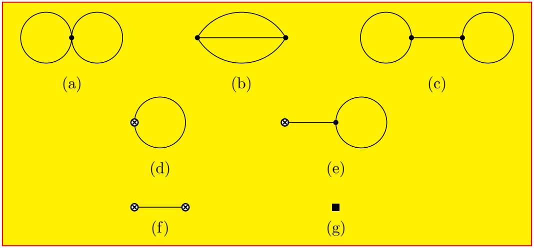
- **This equation yields the matching**

Generating functional

$$\mathbf{Z} = \mathbf{Z_0} + \hbar \mathbf{Z_1} + \hbar^2 \mathbf{Z_2} + \mathbf{O}(\hbar^3),$$

$$\mathbf{Z_0} = \bar{\mathbf{S}}_2\,, \qquad \mathbf{Z_1} = \bar{\mathbf{S}}_4 + \tfrac{1}{2}\,\mathbf{Tr}\,\mathbf{ln}(\mathbf{D}/\mathbf{D^0})\,,$$

$$\mathbf{Z_2} =$$



Generating functional

- **Matching at one-loop order:**

$$\bar{S}^{(3)}_{\text{tree}} + \frac{1}{2}\ln\frac{\det D}{\det D^0} = \bar{s}^{(2)}_{\text{tree}} + \frac{1}{2}\ln\frac{\det d}{\det d^0}$$

- **E.g. for ℓ_6:**

$$\Big(-2L_9 \underbrace{-\frac{1}{12}\int\frac{dq}{(2\pi)^d}\frac{1}{[M_K^2+q^2]^2}}_{\text{from } \det D_K}\Big)\int dx\langle f_{+\mu\nu}[u_\mu, u_\nu]\rangle = \ell_6\int dx\,\underbrace{\langle f_{+\mu\nu}[u_\mu, u_\nu]\rangle}_{\text{chiral operator}}$$

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- **From which one verifies again**

$$-2L_9^r(\mu) + \frac{1}{192\pi^2}(\ln B_0 m_s/\mu^2 + 1) = \ell_6^r(\mu)$$

Generating functional

- **Loops at order $\hbar^2$:**
- **(a) one-particle reducible diagrams, tadpole, butterfly**
- **Steps:**
  - **expand classical actions in quantum fluctations ξ up to necessary order**
  - **by integration by parts put the derivatives to act on the propagators**
  - **extensively use the conservation of strangeness**
  - **use the heat kernel technique to evaluate the Seeley-De Witt coefficients, propagators and their covariant derivatives**
  - **convert appearing monomials into SU(2)-basis**

Generating functional

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  - convert appearing monomials into SU(2)-basis
- **(b) sunset diagram is more difficult to evaluate**
  - need to know the propagators with two covariant derivatives
  - need to expand the Seeley–coefficients around $x = y$ up to 4th order
  - need to express the normal derivatives via the covariant ones (use fixed gauge: courtesy by H.Leutwyler)
  - need to evaluate the tensorial two-loop diagrams of the sunset topology analytically

The results of matching at order of p^2 and p^4

Gasser, Haefeli, Ivanov, Schmid PLB 652 (2007) 21

$$Y = Y_0 \left[1 + a_Y\, x + b_Y\, x^2 + \mathcal{O}(x^3)\right], \qquad Y = F\,, \Sigma,$$

$$\ell_i^r = a_i + x\, b_i + \mathcal{O}(x^2)\,, \qquad i \neq 7\,,$$

$$\ell_7 = \frac{F_0^2}{8 B_0 m_s} + a_7 + x\, b_7 + \mathcal{O}(x^2)\,,$$

$$x = \frac{\overline{M}_K^2}{N F_0^2}\,, \qquad N = 16\pi^2\,, \qquad \Sigma = F^2 B\,, \qquad \Sigma_0 = F_0^2 B_0\,.$$

$\overline{M}_K$ is the one–loop expression of the kaon-mass in the limit $m_u = m_d = 0$.

$a_i \Longrightarrow$ NLO terms (known), $b_i \Longrightarrow$ NNLO terms (obtained).

$$b = p_0 + p_1\, \ell_K + p_2\, \ell_K^2\,, \qquad \ell_K = \ln(\overline{M}_K^2/\mu^2).$$

p_j are independent of the strange quark mass.

The results of matching at order of p^2 and p^4

An example:

$$\ell_2^r = -\frac{1}{24\,N}\,(\ell_K + 1) + 4\,L_2^r$$

$$+ \quad x\left\{\frac{1}{N}\left[\frac{433}{288} - \frac{1}{24}\ln\frac{4}{3} + \frac{1}{16}\,\rho_1 - 16\,N\left(2\,C_{13}^r - C_{11}^r\right)\right]\right.$$

$$\left. + \left[\frac{13}{24} - 8\,L_2^r - 2\,L_3\right]\ell_K + \frac{3}{8}\ell_K^2\right\}$$

$$\rho_1 = \sqrt{2}\,\mathrm{Cl}_2(\arccos(1/3)) \cong 1.41602$$

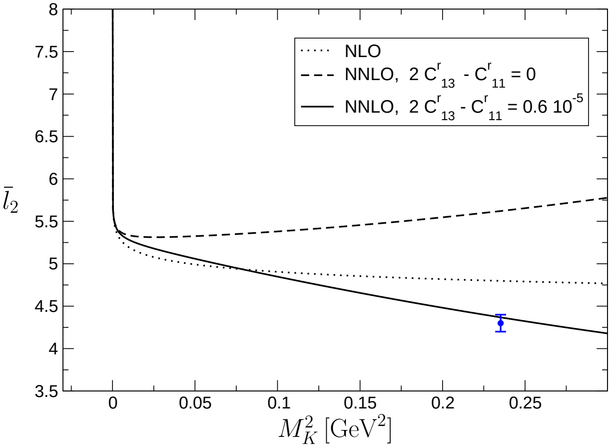
$$\bar{\ell}_2 = 3\,N\,\ell_2^r(\mu) - \ln\frac{M_\pi^2}{\mu^2} = 4.3 \pm 0.1\ ,$$

$$L_2^r = (+0.73 \pm 0.12) \times 10^{-3}, \quad L_3 = (-2.35 \pm 0.37) \times 10^{-3}, \quad \mu = M_\rho,$$

Amoros, Bijnens, Talavera 2001

Large N_c, resonance exchange $\Longrightarrow$ $2\,C_{13}^r - C_{11}^r = 0$ Cirigliano et al. 2006

The results of matching at order of p^2 and p^4



The results of matching at order of p^2 and p^4

Check of the calculations

LEC	Source	Ref.	LEC	Source	Ref.
F	F_π	[1]	F^2B	$\langle 0\|\bar{u}u\|0\rangle$	[2,3,4]
$\ell^r_{1,2}$	$\pi\pi \longrightarrow \pi\pi$	[5]	ℓ^r_3	M_π	[1]
ℓ^r_4	F_π , M_π	[1]	ℓ^r_5	$\langle 0\|A^i_\mu A^k_\nu\|0\rangle$, $\langle 0\|V^i_\mu V^k_\nu\|0\rangle$	[1]
ℓ^r_6	$F_V(t)$	[6]	h^r_1	$\langle 0\|\bar{u}u\|0\rangle$, M_π , F_π	[1,4]
h^r_2	$\langle 0\|V^i_\mu V^k_\nu\|0\rangle$	[1]	h_3	$\langle 0\|S^i S^k\|0\rangle$, B	[1]

[1] G. Amoros, J. Bijnens, P. Talavera, Nucl. Phys. B 568, 319 (2000)

[2] B. Moussallam, JHEP 0008, 005 (2000)

[3] R. Kaiser and J. Schweizer, JHEP 0606, 009 (2006)

[4] J. Bijnens and K. Ghorbani, Phys. Lett. B 636, 51 (2006)

[5] J. Bijnens, P. Dhonte and P. Talavera, JHEP 0401, 050 (2004)

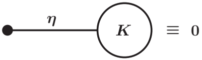
[6] J. Bijnens and P. Talavera, JHEP 0203, 046 (2002)

The results of matching at order of p^6

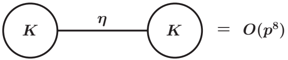
Restricted framework at order of p^6

- **switch off the sources s and p (while retaining m_s)**
- **this yields the following simplifications:**
  - * **reduces about half of the terms in the Lagrangian**
  - * **the solution of the classical EOM for the eta is trivial, $\eta = 0$**
  - * **no mixing between the η and the π^0**
  - * **the one-particle reducible diagrams with eta and kaons do not contribute**

(f)



(e)



(c)

The results of matching at order of p^6

- **The relations among the SU(2)-monomials in the full theory**

Haefeli, Ivanov, Schmid, Ecker 2007

$$
\begin{aligned}
&8P_1 - 2P_2 + 6P_3 - 12P_{13} + 8P_{14} - 3P_{15} - 2P_{16} \\
&-20P_{24} + 8P_{25} + 12P_{26} - 12P_{27} - 28P_{28} + 8P_{36} - 8P_{37} \\
&-8P_{39} + 2P_{40} + 8P_{41} - 8P_{42} - 6P_{43} + 4P_{48} = 0\ .
\end{aligned}
$$

We use this relation to exclude the monomial P_{27}.

The results of matching at order of p^6

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We use this relation to exclude the monomial P_{27}.

- **In the restricted framework, there is an additional relation among the remaining SU(2)–monomials:**

$$\begin{aligned} &8P_1 - 2P_2 + 6P_3 - 20P_{24} + 8P_{25} + 12P_{26} - 16P_{28} - 3P_{29} \\ &+ 3P_{30} - 6P_{31} + 12P_{32} - 3P_{33} + 8P_{36} - 8P_{37} - 11P_{39} \\ &+ 5P_{40} + 14P_{41} - 8P_{42} - 9P_{43} + 3P_{44} - 3P_{45} - 6P_{51} - 6P_{53} = 0\,. \end{aligned}$$

We use this relation to exclude the monomial P_1.

The results of matching at order of p^6

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$$x_i = p_i^{(0)} + p_i^{(1)}\ell_K + p_i^{(2)}\ell_K^2 + O(m_s)$$

i	x_i	i	x_i	i	x_i
1	$c_2^r + \frac{1}{4}c_1^r$	10	$c_{32}^r - \frac{3}{2}c_1^r - c_{27}^r$	19	$c_{43}^r + \frac{9}{8}c_1^r + \frac{1}{4}c_{27}^r$
2	$c_3^r - \frac{3}{4}c_1^r$	11	$c_{33}^r + \frac{3}{8}c_1^r + \frac{1}{4}c_{27}^r$	20	$c_{44}^r - \frac{3}{8}c_1^r - \frac{1}{4}c_{27}^r$
3	$c_{24}^r + \frac{5}{2}c_1^r$	12	$c_{36}^r - c_1^r$	21	$c_{45}^r + \frac{3}{8}c_1^r + \frac{1}{4}c_{27}^r$
4	$c_{25}^r - c_1^r$	13	$c_{37}^r + c_1^r$	22	c_{50}^r
5	$c_{26}^r - \frac{3}{2}c_1^r$	14	c_{38}^r	23	$c_{51}^r + \frac{3}{4}c_1^r + \frac{1}{2}c_{27}^r$
6	$c_{28}^r + 2c_1^r - c_{27}^r$	15	$c_{39}^r + \frac{11}{8}c_1^r + \frac{1}{4}c_{27}^r$	24	c_{52}^r
7	$c_{29}^r + \frac{3}{8}c_1^r + \frac{1}{4}c_{27}^r$	16	$c_{40}^r - \frac{5}{8}c_1^r - \frac{1}{4}c_{27}^r$	25	$c_{53}^r + \frac{3}{4}c_1^r + \frac{1}{2}c_{27}^r$
8	$c_{30}^r - \frac{3}{8}c_1^r - \frac{1}{4}c_{27}^r$	17	$c_{41}^r - \frac{7}{4}c_1^r - \frac{1}{2}c_{27}^r$	26	c_{55}^r
9	$c_{31}^r + \frac{3}{4}c_1^r + \frac{1}{2}c_{27}^r$	18	$c_{42}^r + c_1^r$	27	c_{56}^r

Two checks of our calculations at order of p^6

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- **(1) The vector-vector correlator** (Amoros, Bijnens, Talavera 2000)

$$c^r_{56} = -\frac{1}{240}\frac{F^2}{N\overline{M}_K^2} - \frac{1}{288N^2} + \frac{1}{6N}L^r_9 + C^r_{93} - \left(\frac{1}{144N^2} - \frac{1}{6N}L^r_9\right)\ell_K - \frac{1}{288N^2}\ell_K^2$$

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- **(2) The pion form factor** (Bijnens, Colangelo, Talavera 1998, 2002)

$$\begin{aligned} c^r_{51} - c^r_{53} &= +\frac{319}{73728N^2} - \frac{1}{480N}\frac{F^2}{N\overline{M}_K^2} + \frac{245}{98304N^2}\ln\frac{4}{3} \\ &\quad - \frac{1}{12N}L^r_3 + \frac{1}{24N}L^r_9 + C^r_{88} - C^r_{90} + \frac{301}{196608N^2}\rho_1 \\ &\quad + \left(\frac{7}{1728N^2} - \frac{1}{12N}L^r_3 + \frac{1}{24N}L^r_9\right)\ell_K - \frac{1}{1152N^2}\ell_K^2 \end{aligned}$$

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\end{aligned}
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- **Matching of the order p^6 LECs in the parity-odd sector was performed by K. Kampf and B. Moussallam (arXiv:0901.4688 [hep-ph].)**

Summary

- We studied $\chi\mathrm{PT}_3$ in the limit $m_u, m_d \ll m_s\ ,\ \ |p^2| \ll M_K^2\ ,$ and assuming that the external sources live in the two-flavor subspace, e.g. $v_\mu = \sum_{i=1}^{3} v_\mu^i \lambda^i$.

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