SOME ASPECTS OF ISOSPIN VIOLATION IN KAON DECAYS

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$K_{\ell 3}$ decays

Fully inclusive decay rate $\Gamma(K_{\ell 3[\gamma]})$

$$\Gamma = \frac{G_F^2\,|V_{us}|^2\,M_K^5\,C_K^2}{128\,\pi^3}\,S_{ew}\,|f_+^{K^0\pi^-}(0)|^2\,I_{K\ell}^{(0)}(\lambda_i)\,\left(1+\delta_{EM}^{K\ell}+\delta_{SU(2)}^{K\pi}\right)$$

$$C_K = \begin{cases} 1 & \text{for } K_{\ell 3}^0 \\ 1/\sqrt{2} & \text{for } K_{\ell 3}^+ \end{cases}$$

$$\delta_{EM}^{K\ell} = \delta_{EM}^{K\ell}(\mathcal{D}_3) + \delta_{EM}^{K\ell}(\mathcal{D}_{4-3}), \qquad \delta_{SU(2)}^{K\pi} = \left(\frac{f_+^{K\pi}(0)}{f_+^{K^0\pi^-}(0)}\right)^2 - 1$$

Electromagnetic corrections

Short distance electroweak corrections

$$S_{\mathrm{ew}} = 1 + \frac{2\alpha}{\pi}\left(1 - \frac{\alpha_s}{4\pi}\right) \times \log\frac{M_Z}{M_\rho} + \mathcal{O}\left(\frac{\alpha\alpha_s}{\pi^2}\right) = 1.0223 \pm 0.0005$$

universal factor **Sirlin 1978, 1982**

Long distance EM corrections

appropriate EFT : CHPT with virtual photons and leptons

Knecht, N., Rupertsberger, Talavera 2000

general formulae for $K_{\ell 3}$ EM corrections

Cirigliano, Knecht, N., Rupertsberger, Talavera 2002

numerics for K_{e3} Cirigliano, N., Pichl 2004

- ☛ numerics for K_{e3} (update) and $K_{\mu 3}$ (new) Cirigliano, Giannotti, N. 2008
- ✷ update of structure-dependent EM contributions
  (K_i^r from Ananthanarayan, Moussallam 2004
  and X_i^r from Descotes-Genon, Moussallam 2005)

Numerical results

	$I^{(0)}_{K\ell}(\lambda_i)$	$\delta^{K\ell}_{\rm EM}(\mathcal{D}_3)(\%)$	$\delta^{K\ell}_{\rm EM}(\mathcal{D}_{4-3})(\%)$	$\delta^{K\ell}_{\rm EM}(\%)$
K^0_{e3}	0.103070	0.50	0.49	0.99 ± 0.22
$K^\pm_{e3}$	0.105972	-0.35	0.45	0.10 ± 0.25
$K^0_{\mu 3}$	0.068467	1.38	0.02	1.40 ± 0.22
$K^\pm_{\mu 3}$	0.070324	0.007	0.009	0.016 ± 0.25

errors: estimates of higher-order contributions

“Soft-photon factorization”

includes incomplete higher order terms in the chiral expansion

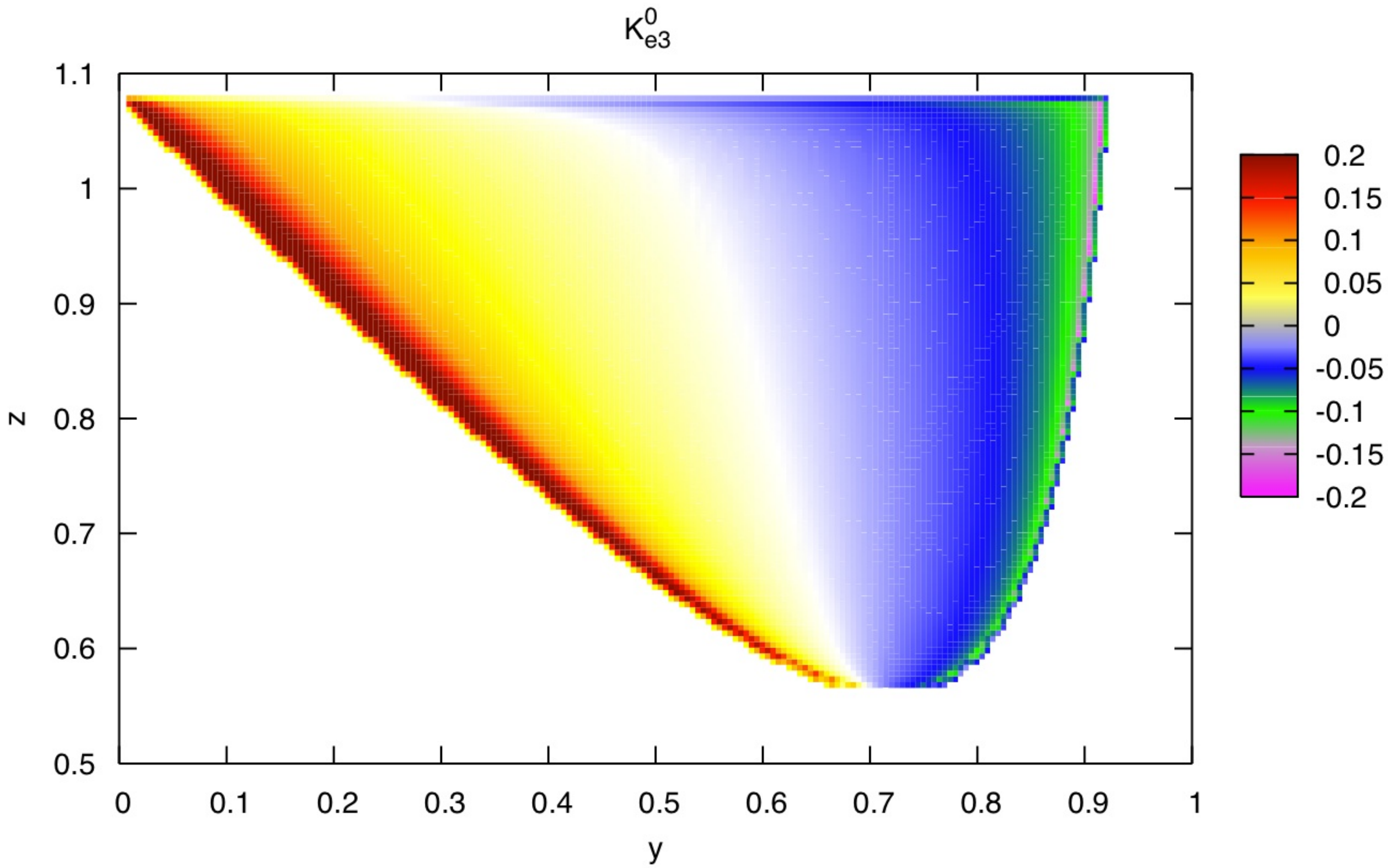
	$\delta_{\mathrm{EM}}^{K\ell}(\mathcal{D}_3)(\%)$	$\delta_{\mathrm{EM}}^{K\ell}(\mathcal{D}_{4-3})(\%)$	$\delta_{\mathrm{EM}}^{K\ell}(\%)$
K_{e3}^0	0.41	0.59	1.0
$K_{e3}^{\pm}$	-0.564	0.528	-0.04
$K_{\mu3}^0$	1.57	0.04	1.61
$K_{\mu3}^{\pm}$	-0.006	0.011	0.005

$\longrightarrow$ **validates estimates of theoretical uncertainties**

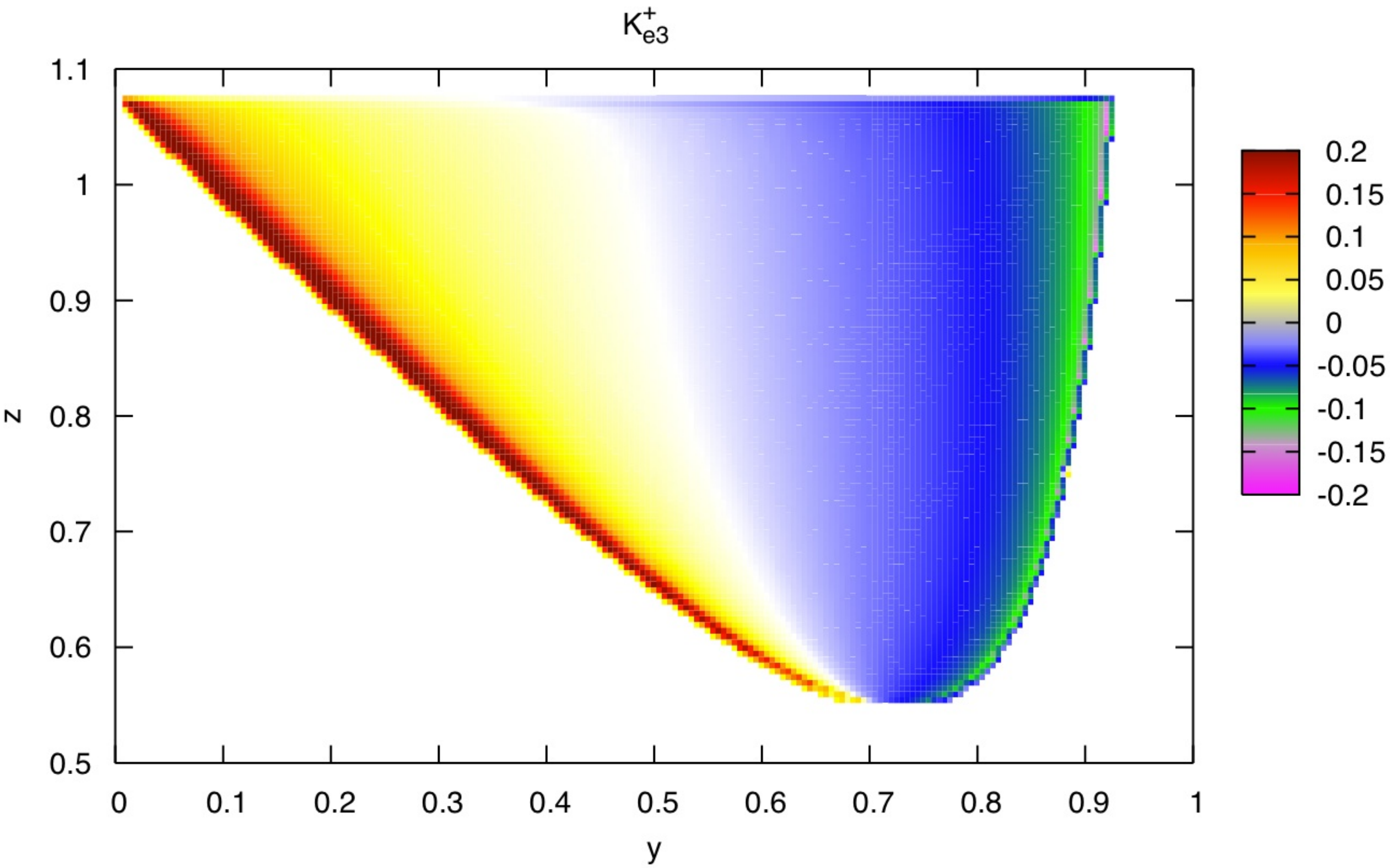
Decay distribution with EM corrections

$$\frac{d\Gamma}{dy\,dz} = \frac{G_F^2\,|V_{us}|^2\,M_K^5\,C_K^2}{128\,\pi^3}\,S_{\rm ew}\,|f_+^{K\pi}(0)|^2\left[\bar\rho^{(0)}(y,z)\;+\;\delta\bar\rho^{\rm EM}(y,z)\right]$$

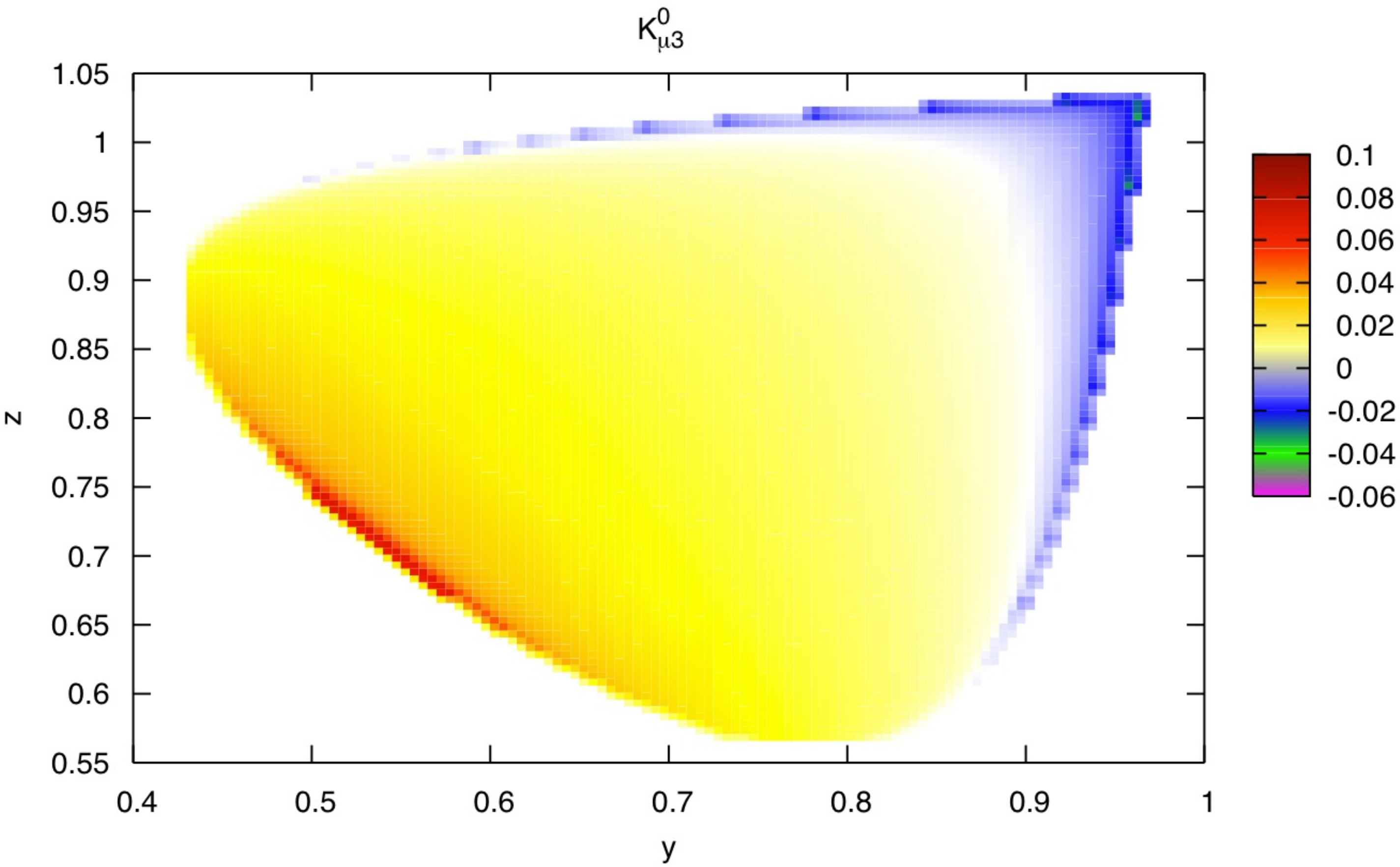
$$z = \frac{2p_\pi\cdot p_K}{M_K^2} = \frac{2E_\pi}{M_K}\,,\qquad y = \frac{2p_K\cdot p_\ell}{M_K^2} = \frac{2E_\ell}{M_K}$$



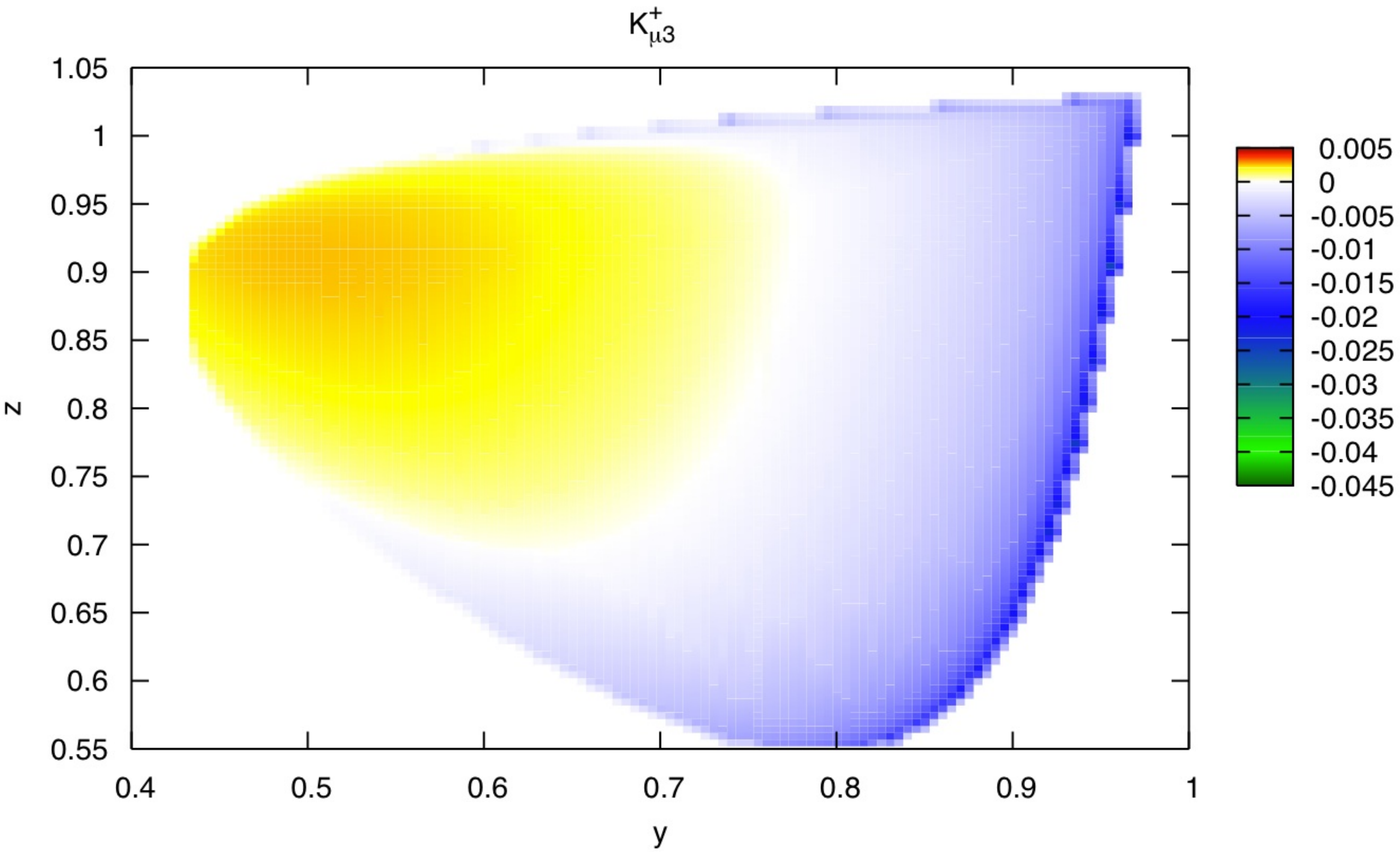
ratio $\delta\bar{\rho}^{\mathrm{EM}}(y,z)/\bar{\rho}^{(0)}(y,z)$ for K^0_{e3}



ratio $\delta\bar{\rho}^{\mathrm{EM}}(y,z)/\bar{\rho}^{(0)}(y,z)$ for K_{e3}^{+}



ratio $\delta\bar{\rho}^{\mathrm{EM}}(y,z)/\bar{\rho}^{(0)}(y,z)$ for $K^0_{\mu 3}$



ratio $\delta\bar{\rho}^{\mathrm{EM}}(y,z)/\bar{\rho}^{(0)}(y,z)$ for $K^+_{\mu 3}$

Determination of $\delta^{K\pi}_{\mathrm{SU(2)}}$

$$\delta^{K\pi}_{\mathrm{SU(2)}} = \begin{cases} 0 & \text{for } K^0_{\ell 3} \\ 2\sqrt{3}\left(\varepsilon^{(2)} + \varepsilon^{(4)}_{\mathrm{S}} + \varepsilon^{(4)}_{\mathrm{EM}} + \dots\right) & \text{for } K^+_{\ell 3} \end{cases}$$

$$\varepsilon^{(2)} = \frac{\sqrt{3}}{4}\frac{m_d - m_u}{m_s - \widehat{m}} \qquad \widehat{m} = \frac{m_u + m_d}{2}$$

$\longrightarrow$ **need determination of quark mass ratio**

$$R := \frac{m_s - \widehat{m}}{m_d - m_u}$$

double ratio

$$Q^2 := \frac{m_s^2 - \widehat{m}^2}{m_d^2 - m_u^2} = R\,\frac{m_s/\widehat{m} + 1}{2}$$

can be expressed in terms of meson masses and a purely EM contribution

Gasser, Leutwyler 1985

$$Q^2 = \frac{\Delta_{K\pi} M_K^2 \left(1 + \mathcal{O}(m_q^2)\right)}{M_\pi^2 \left[\Delta_{K^0K^+} + \Delta_{\pi^+\pi^0} - (\Delta_{K^0K^+} + \Delta_{\pi^+\pi^0})_{\mathrm{EM}}\right]}, \qquad \Delta_{PQ} = M_P^2 - M_Q^2$$

$(\Delta_{K^0K^+} + \Delta_{\pi^+\pi^0})_{\mathrm{EM}}$ **vanishes to lowest order** e^2p^0 **Dashen 1969**

$$(\Delta_{K^0K^+} + \Delta_{\pi^+\pi^0})_{\mathrm{EM}} = e^2 M_K^2 \left[\frac{1}{4\pi^2}\left(3\ln\frac{M_K^2}{\mu^2} - 4 + 2\ln\frac{M_K^2}{\mu^2}\right) + \frac{4}{3}(K_5 + K_6)^r(\mu) - 8(K_{10} + K_{11})^r(\mu) + 16 Z L_5^r(\mu)\right] + \mathcal{O}(e^2 M_\pi^2)$$

Urech 1995; N., Rupertsberger 1995

Ananthanarayan, Moussallam 2004: large deviation from Dashen's limit

$$(\Delta_{K^0K^+} + \Delta_{\pi^+\pi^0})_{\text{EM}} = -1.5\,\Delta_{\pi^+\pi^0} \quad \longrightarrow \quad Q = 20.7 \pm 1.2$$

$Q = 22.7 \pm 0.8$ **Leutwyler 1996**

$Q = 22.0 \pm 0.6$ **Bijnens, Prades 1997**

$Q \simeq 20$ **Amoros, Bijnens, Talavera 2001**

however: $Q = 23.2$ **($\eta \to 3\pi$ at two loops)** **Bijnens, Ghorbani 2007**

determinations of second input parameter $m_s/\widehat{m} \sim 24$ rather stable

$$\left.\begin{array}{l} Q = 20.7 \pm 1.2 \\ m_s/\widehat{m} = 24.7 \pm 1.1 \end{array}\right\} \longrightarrow \; R = 33.5 \pm 4.3 \;\longrightarrow\; \delta_{\text{SU(2)}} = 0.058(8)$$

Kastner, N., 2008

$\delta_{\text{SU(2) exp.}} = 0.058(8)$ **FLAVIAnet Working Group 2008**

$K_{\ell 3}$ scalar form factor

$$f_0^{K\pi}(t) = f_+^{K\pi}(t) + \frac{t}{M_K^2 - M_\pi^2} f_-^{K\pi}(t) \quad \Rightarrow \quad f_0^{K\pi}(0) = f_+^{K\pi}(0)$$

Slope parameter, curvature

$$\frac{f_0^{K\pi}(t)}{f_+^{K\pi}(0)} = 1 + \lambda_0^{K\pi} \frac{t}{M_{\pi^+}^2} + \frac{1}{2} c_0^{K\pi} \left(\frac{t}{M_{\pi^+}^2}\right)^2 + \ldots$$

Experimental results for $\lambda_0^{K\pi}$

ISTRA+	KTeV	NA48	KLOE
0.0171(22)	0.0137(13)	0.0095(14)	0.0154(22)

ISTRA+: $K^- \to \pi^0 \mu^- \nu$, **KTeV, NA48, KLOE:** $K^0_{L\mu 3}$

ISTRA+ $\leftrightarrow$ **NA48** **gigantic isospin breaking?**

KTeV $\leftrightarrow$ **NA48** $\leftrightarrow$ **KLOE** **consistent?**

NA48: **Callan Treiman?**

Slopes at order p^4, $(m_d - m_u)p^2$, e^2p^2

$$\lambda_0^{K^0\pi^-} = (\underbrace{16.64}_{m_u=m_d} + \underbrace{0.17}_{m_u\neq m_d} + \underbrace{0.14}_{\mathrm{EM}}) \times 10^{-3}$$
$$= (16.95 \pm 0.40_{F_K/F_\pi f_+(0)} \pm 0.05_{\varepsilon^{(2)}}) \times 10^{-3}$$

$$\lambda_0^{K^+\pi^0} = (\underbrace{16.64}_{m_u=m_d} - \underbrace{0.12}_{m_u\neq m_d} - \underbrace{0.08}_{\mathrm{EM}}) \times 10^{-3}$$
$$= (16.44 \pm 0.39_{F_K/F_\pi f_+(0)} \pm 0.04_{\varepsilon^{(2)}}) \times 10^{-3}$$

$$\longrightarrow \Delta\lambda_0 := \lambda_0^{K^0\pi^-} - \lambda_0^{K^+\pi^0} = (5.1 \pm 0.9) \times 10^{-4}$$

Analysis at NNLO (isospin limit)

large shift: $\lambda_0^{K\pi} = \left(13.9^{+1.3}_{-0.4} \pm 0.4\right) \times 10^{-3}$ **Kastner, N. 2008**

combines two-loop result **Bijnens, Talavera 2003**

and large N_c estimate of LECs $C_{12},\ C_{34}$

Cirigliano, Ecker, Eidemüller, Kaiser, Pich, Portolés 2005

Contributions of order $(m_d - m_u)p^4$

extracted from Bijnens, Ghorbani (2007): $\Delta\lambda_0\big|_{C_i^r=e=0} \simeq 5 \times 10^{-4}$

contribution of LECs:

$$\Delta\lambda_0\big|_{C_i^r} = \frac{32\varepsilon^{(2)}\Delta_{K\pi}M_{\pi^+}^2}{\sqrt{3}F_\pi^4}\left(2C_{12}+6C_{17}+6C_{18}+3C_{34}+3C_{35}\right)^r(M_\rho)$$

using list of LECs given by Cirigliano, Ecker, Eidemüller, Kaiser, Pich, Portolés (2006)**:**

$$\left(2C_{12}+6C_{17}+6C_{18}+3C_{34}+3C_{35}\right)^{\mathcal{SP}} = \frac{F_\pi^4}{4M_S^4}\left(1-\frac{3M_S^2}{2M_P^2}-\frac{M_S^2}{M_{\eta'}^2}+6\lambda_2^{\mathcal{SS}}\right)$$

$$\left|\lambda_2^{\mathcal{SS}}\right| \lesssim 1 \quad \longrightarrow \quad 0 \lesssim \Delta\lambda_0 \lesssim 10^{-3}$$

Summary

- CHPT suitable framework for EM corrections in semileptonic decays
- theoretical estimates for all electromagnetic LECs $K_i^r,\ X_i^r$
- EM corrections for all K_{l3} decay modes
- proper treatment of EM corrections mandatory in analysis of $K_{\ell 3}$ data
- (probably) large deviation from Dashen's limit $\longrightarrow$ influence on $\delta^{K\pi}_{\mathrm{SU(2)}}$
- Isospin violation increases the uncertainty of the determination of the scalar slope parameters by (at most) $\pm 10^{-3}$ with $0 \lesssim \lambda_0^{K^0\pi^-} - \lambda_0^{K^+\pi^0} \lesssim 10^{-3}$
- results of ISTRA+, KTeV and KLOE are in agreement with the SM prediction