Light quark masses

H. Leutwyler

University of Bern

Chiral Dynamics, July 6 2009

Standard Model at low energies

- Low energies ($E \ll M_{\text{W}}$): weak interaction is frozen

⇒ Standard Model reduces to QCD + QED

- Lagrangian only involves g, θ, e, fermion masses

⇒ Precision theory for cold matter ($T \ll M_{\text{W}}$), size and structure of atoms, solids, etc.

- QED is infrared stable, characterized by pure number, which happens to be small, $1/137$

⇒ QED can be accounted for with perturbation theory

- At low energies: SM = QCD + corrections

Chiral symmetry

- QCD with N_f massless quarks: Hamiltonian has an exact symmetry, $\mathrm{SU_L}(N_f)\times\mathrm{SU_R}(N_f)$
- $|0\rangle$ is symmetric only under the subgroup $\mathrm{SU_{L+R}}(N_f)$
  Symmetry is hidden, "spontaneously broken"

$\Rightarrow$ Spectrum contains N_f^2-1 Goldstone bosons

- m_{u} and m_{d} happen to be small

$\Rightarrow$ $\mathrm{SU_L}(2)\times\mathrm{SU_R}(2)$ is an approximate symmetry of QCD

  - broken spontaneously: $|0\rangle$ not invariant
  - broken explicitly: $\mathcal{L}_{\mathrm{QCD}}$ not invariant
    Symmetry broken by mass term $m_{\mathrm{u}}\,\bar{u}u + m_{\mathrm{d}}\,\bar{d}d$,
    but since $m_{\mathrm{u}}, m_{\mathrm{d}}$ are small, the breaking is weak

Hidden symmetries in particle physics

Already in 1960, Nambu realized that

1. $SU_L(2)\times SU_R(2)$ is an approximate symmetry of the strong interaction
2. The symmetry is spontaneously broken:
   $|0\rangle$ invariant only under the isospin subgroup SU(2)
3. The spontaneous breakdown of an exact symmetry entails massless particles
4. For the strong interaction, the pions play this role
5. The pions are not massless, only light, because the symmetry is only an approximate one

Nobel Prize 2008

Explains why the energy gap of the strong interaction is so small : $M_\pi \simeq$ 140 MeV
When Nambu proposed this idea, the origin of the symmetry was mysterious
Approximate symmetries ? Partially conserved currents ?
For gauge theories like QCD, approximate chiral symmetries do occur naturally

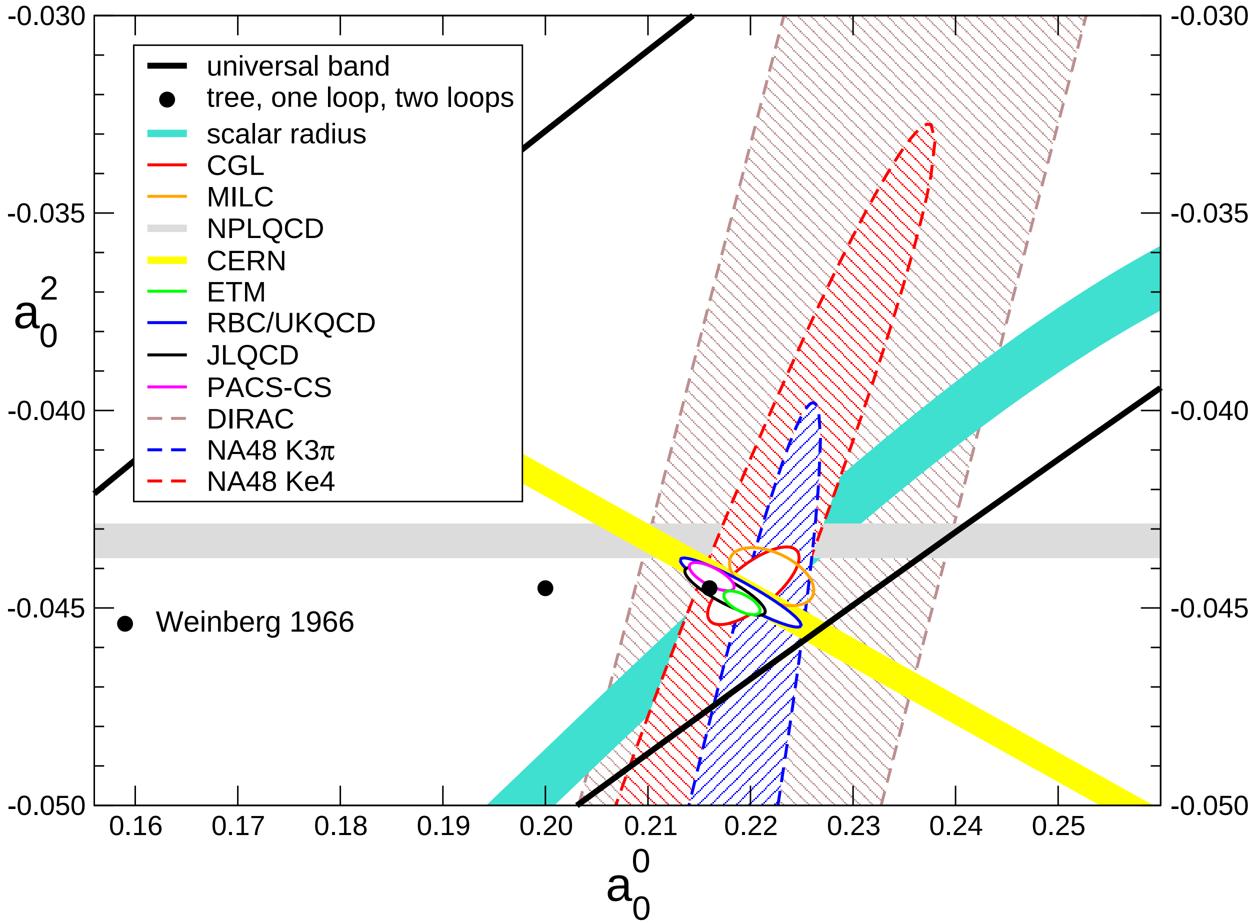
Chiral perturbation theory based on SU(2)$\times$SU(2)

- Expansion in powers of $m_\mathrm{u}, m_\mathrm{d}$ yields a very accurate low energy representation of QCD

Low energy pion physics is a precision laboratory Theoretical tools: χPT, lattice, dispersion theory

- Limitations:
  - Low energies
  - e.m. interaction must properly be accounted for
  - Calculations cannot be done on back of an envelope

Illustration: $\pi\pi$ scattering lengths



Lattice results for $\bar{\ell}_3, \bar{\ell}_4$ are translated into values for a_0^0, a_0^2

Contributions from higher order couplings are tiny

Guo + Sanz-Cillero arXiv:0904.4178

Extension to SU(3)×SU(3)

- In the theoretical limiting case $m_\mathrm{u} = m_\mathrm{d} = m_\mathrm{s} = 0$ QCD acquires an exact SU(3)$_\mathrm{L}$×SU(3)$_\mathrm{R}$ symmetry

Is m_s small enough for this to represent a useful approximate symmetry ?

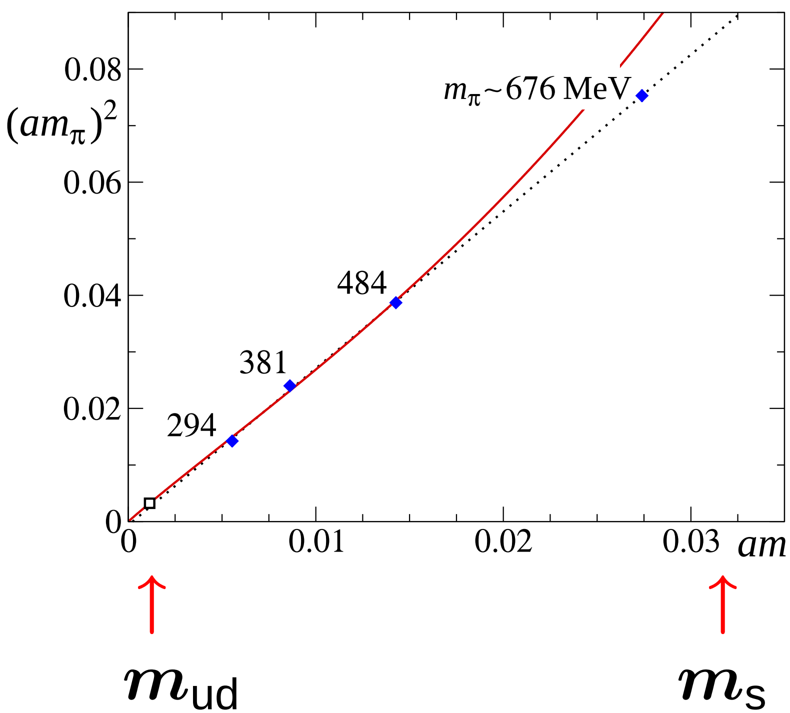
- Theoretical reasoning
  - SU(3)$_\mathrm{L+R}$ (eightfold way) is an approximate symmetry
  - Typical size of SU(3)$_\mathrm{L+R}$ breaking: $\dfrac{F_K}{F_\pi} = 1.19 \pm 0.01$
  - Only coherent way to understand this in QCD:

    The mass differences $m_\mathrm{s} - m_\mathrm{d}$, $m_\mathrm{d} - m_\mathrm{u}$ must be small, can be treated as perturbations
  - Since $m_\mathrm{u}, m_\mathrm{d} \ll m_\mathrm{s}$

$\Rightarrow$ m_s is small, SU(3)$_\mathrm{L}$×SU(3)$_\mathrm{R}$ must be an approximate symmetry, breaking not larger than for SU(3)$_\mathrm{L+R}$

Expansion in powers of $m_{\rm u}, m_{\rm d}, m_{\rm s}$

- Expansion in powers of $m_{\rm u}, m_{\rm d}, m_{\rm s}$ ought to work, but expect convergence to be comparatively slow
  $m_{\rm ud} \equiv \frac{1}{2}(m_{\rm u} + m_{\rm d})$
- Lattice results: $M_\pi^2 \propto m_{\rm ud}$ holds out to $10 \times m_{\rm ud}^{\rm phys}$
- $m_{\rm s}$ is larger than that: $m_{\rm s} \simeq 27 \times m_{\rm ud}$



Compare

$$\frac{F_K}{F_\pi} \simeq 1.19$$

Lüscher, 2005

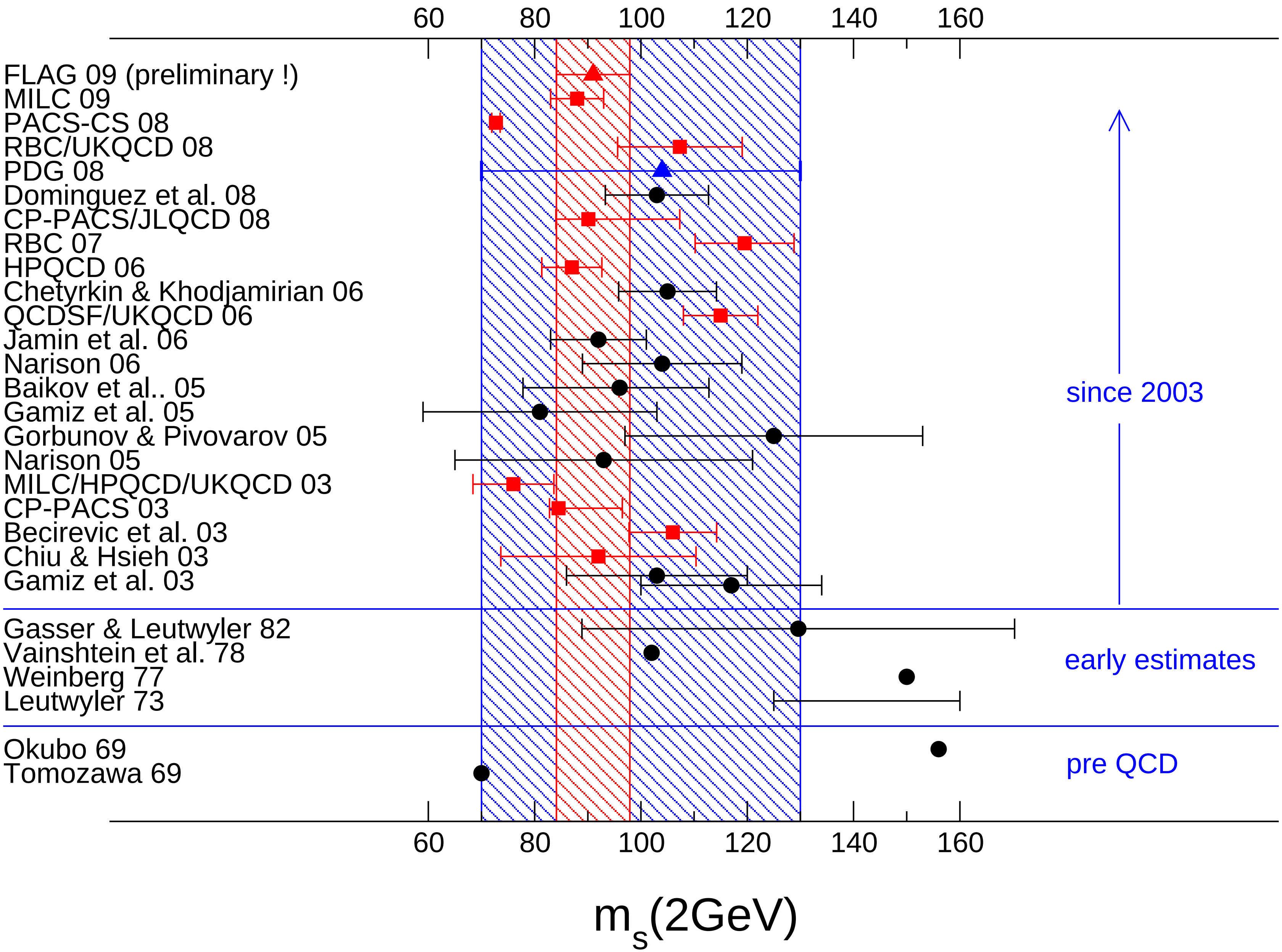
Three light quarks: interface between lattice and χPT

- Steady progress in simulating QCD with light quarks
- Still, the quark masses used are too large for the NLO formulae of χPT to work
- M_π OK, but M_K too large
- Three options
  - Use smaller quark masses
  - Extrapolate only in m_u, m_d, keep m_s fixed
  - Account for NNLO contributions
- Some lattice analyses do allow for NNLO contributions, but the chiral logarithms are accounted for only to NLO
- $\exists$ discrepancies between different lattice results

  In part, these may arise from nonperturbative renormalization effects
  Some of the collaborations still use perturbative renormalization

$\Rightarrow$ Illustrate this with the results for m_s

Mass of the strange quark



Conclusion for m_s

- Lattice and sum rule results agree within errors
- Uncertainties in lattice determinations steadily become smaller, will decrease further

Concerning the relative size of the light quark masses, the situation is somewhat less satisfactory – I now turn to that

Mass formulae at tree level of χPT

- $$M^2_{\pi^+} = (m_u + m_d)\, B_0 + O(m^2)$$
  $$M^2_{K^+} = (m_u + m_s)\, B_0 + O(m^2)$$
  $$M^2_{K^0} = (m_d + m_s)\, B_0 + O(m^2)$$
- χPT relates B_0 to the quark condensate, but does not predict its size $\Rightarrow$ no prediction for size of quark masses
- Account for e.m. self energies at tree level of χPT and drop effects of second order in isospin breaking
  $$\frac{m_u}{m_d} = \frac{M^2_{K^+} - M^2_{K^0} + 2M^2_{\pi^0} - M^2_{\pi^+}}{M^2_{K^0} - M^2_{K^+} + M^2_{\pi^+}} = 0.56$$
  $$\frac{m_s}{m_d} = \frac{M^2_{K^+} + M^2_{K^0} - M^2_{\pi^+}}{M^2_{K^0} - M^2_{K^+} + M^2_{\pi^+}} = 20.2$$
  Weinberg 1977
- Corrections from higher orders ? Could they strongly modify these numerical values ? m_u = 0 ?

$m_u = 0$?

- Suppose m_u vanishes. The formula for m_u/m_d then turns into a prediction for $M_{K^0} - M_{K^+}$:

$$M_{K^0} - M_{K^+} = \frac{2M^2_{\pi^0} - M^2_{\pi^+}}{M_{K^0} + M_{K^+}} \left\{ 1 + O[m] \right\}$$

$m_u = 0$?

- Suppose m_u vanishes. The formula for m_u/m_d then turns into a prediction for $M_{K^0} - M_{K^+}$:

$$M_{K^0} - M_{K^+} = \frac{2M^2_{\pi^0} - M^2_{\pi^+}}{M_{K^0} + M_{K^+}} \left\{ 1 + O[m] \right\}$$

$\Uparrow$ 3.9 MeV $\qquad$ $\Uparrow$ 16.9 MeV

$\Rightarrow$ If m_u vanishes then χPT fails:

- chiral series cannot be truncated at low orders
- $SU(3)_L \times SU(3)_R$ not an approximate symmetry
- Gell-Mann-Okubo formula an accident, etc.

- Very generous range for which a truncation of the chiral expansion is halfway legitimate:

$$0.25 < m_u/m_d < 0.7$$

Corrections of NLO

- Work with the ratios S and Q

$$S \equiv \frac{m_{\rm s}}{m_{\rm ud}} = \frac{2M_K^2}{M_\pi^2}\left\{1-\Delta_{\rm M}\right\} - 1$$

$$Q^2 \equiv \frac{m_{\rm s}^2 - m_{\rm ud}^2}{m_{\rm d}^2 - m_{\rm u}^2} = \frac{M_K^2 - M_\pi^2}{M_{K^0}^2 - M_{K^+}^2}\,\frac{M_K^2}{M_\pi^2}\left\{1-\Delta_{\rm Q}\right\}$$

- Remarkably, the second one does not receive a correction at NLO: $\Delta_{\rm Q} = O[m^2, e^2]$ Gasser & L. 1985
- Insert Weinberg's leading order ratios $\Rightarrow$ Q = 24.3.

$\Rightarrow$ In the plane of $m_{\rm s}/m_{\rm d}$ versus $m_{\rm u}/m_{\rm d}$, a given value of Q corresponds to an ellipse

- Critical input here is the "Dashen theorem": e.m. self energies are accounted for only at tree level

$\eta \to \pi^+\pi^-\pi^0$

- η decay allows an independent determination of Q

  Gasser & L. 1985

- In this transition, the e.m. contributions are suppressed

  Bell & Sutherland 1968

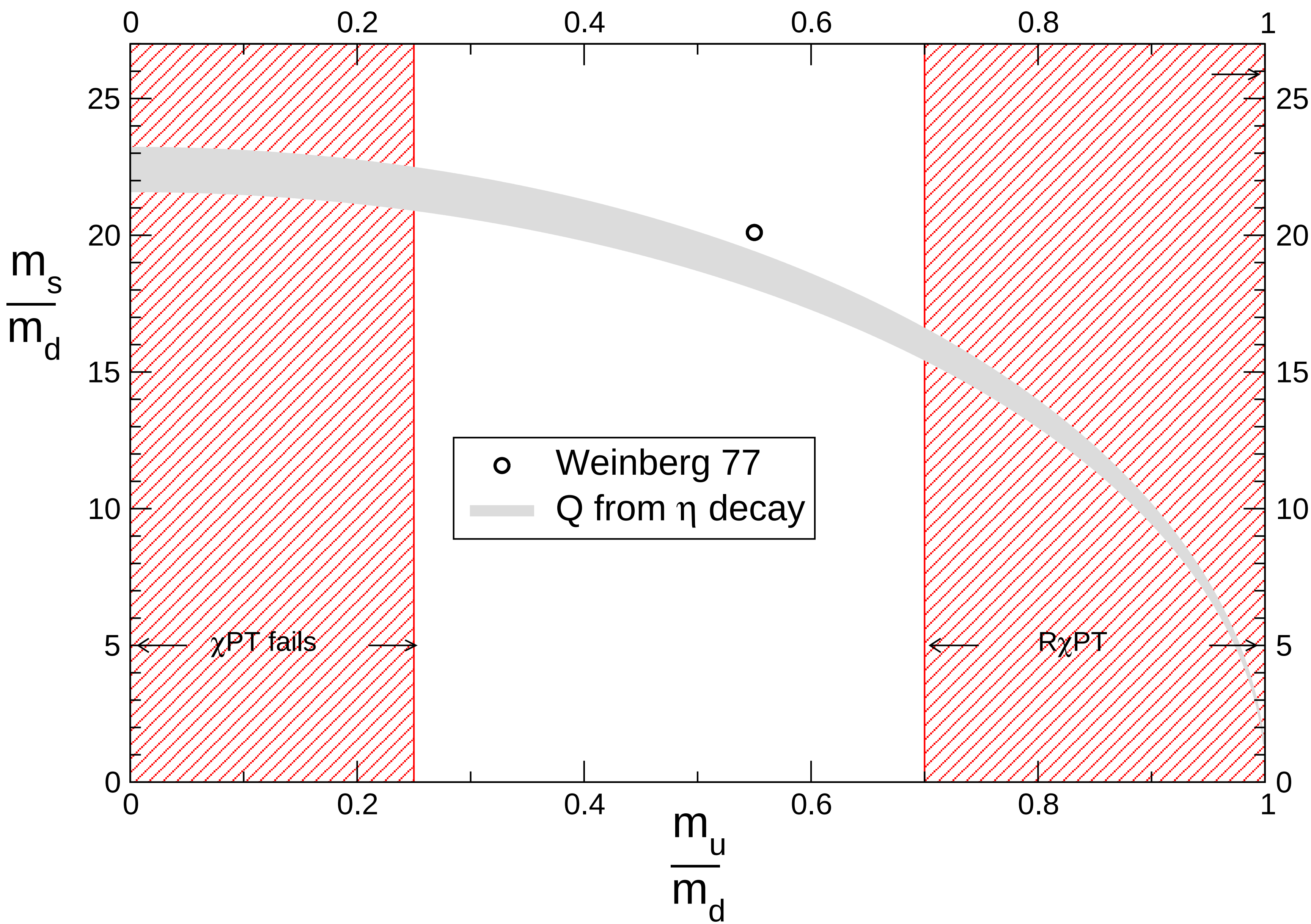
- Dispersive analysis of the decay amplitude

  Kambor, Wiesendanger & Wyler 1996, Anisovich & L. 1996, Walker 1998

  $\Rightarrow$ talk by Stefan Lanz in WG 1

- Update of Walker's calculation with the current experimental information $\Rightarrow$ Q = 22.4 $\pm$ 0.8, to be compared with Q = 24.3 from Dashen theorem

Allowed range of mass ratios



Where on the ellipse ?

- $\nexists$ scalar probe analogous to γ, $W^{\pm}$

$\Rightarrow$ Not all effective coupling constants can be determined from phenomenology alone

$\Rightarrow$ Position on ellipse cannot be determined from phenomenology alone
Kaplan & Manohar 1986

- In particular, all determinations of the ratio

$$R \equiv \frac{m_{\rm s}-m_{\rm ud}}{m_{\rm d}-m_{\rm u}} = \frac{2\,Q^2}{S+1}$$

compares breaking of SU(3) and SU(2)

face this problem:

  - Isospin breaking in other multiplets
  - $\rho - \omega$ mixing
  - $\Gamma_{\psi'\to\psi\pi^0}/\Gamma_{\psi'\to\psi\eta}$

Large N_c

- Problem disappears in the large N_c limit
- In this limit, the η' also becomes a Goldstone boson

$\Rightarrow$ Can extend χPT to include the η', systematic expansion in powers of $m_{\rm u}$, $m_{\rm d}$, $m_{\rm s}$ and $1/N_c$

- In this framework, there is no ambiguity at NLO
- Triangle anomaly yields a prediction also for $\Gamma_{\eta' \to \gamma\gamma}$
  Can use this to pin down all unknowns at NLO

Kaiser 1997

η and η' at large N_c

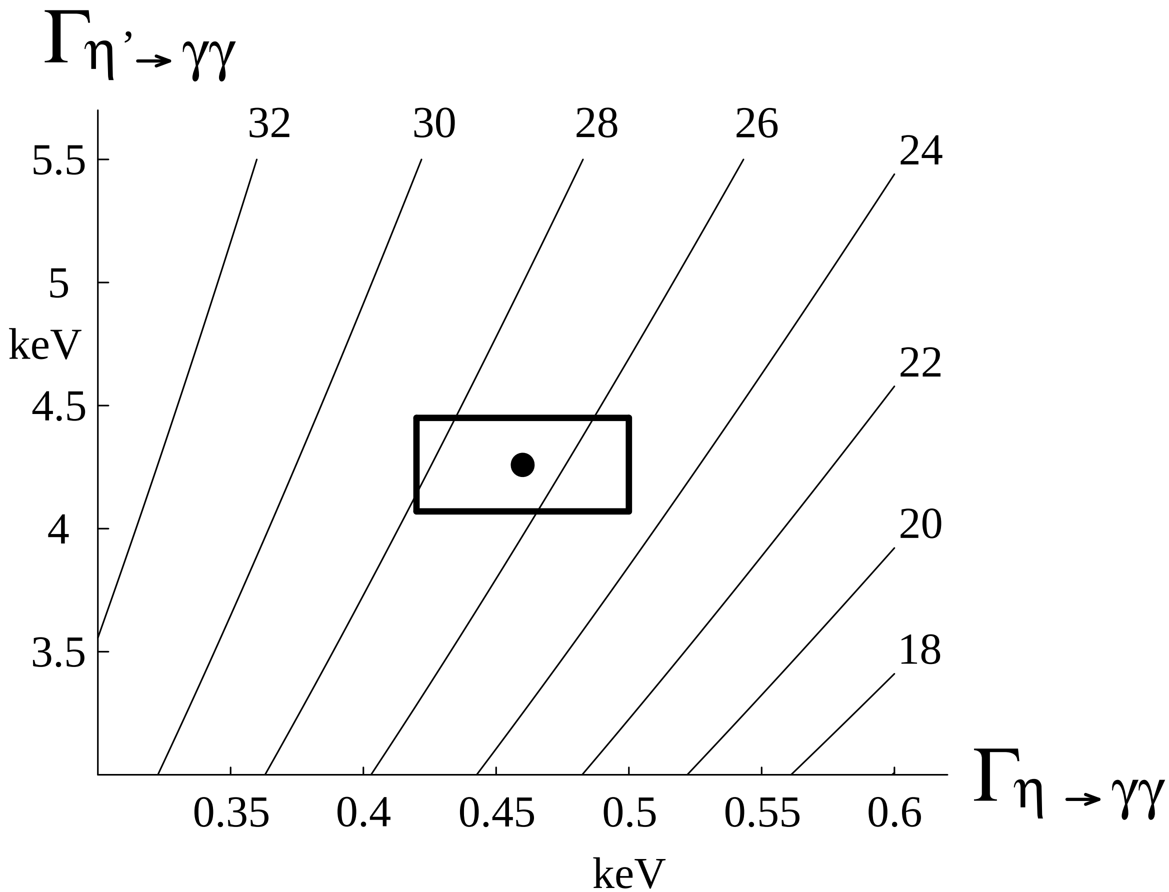


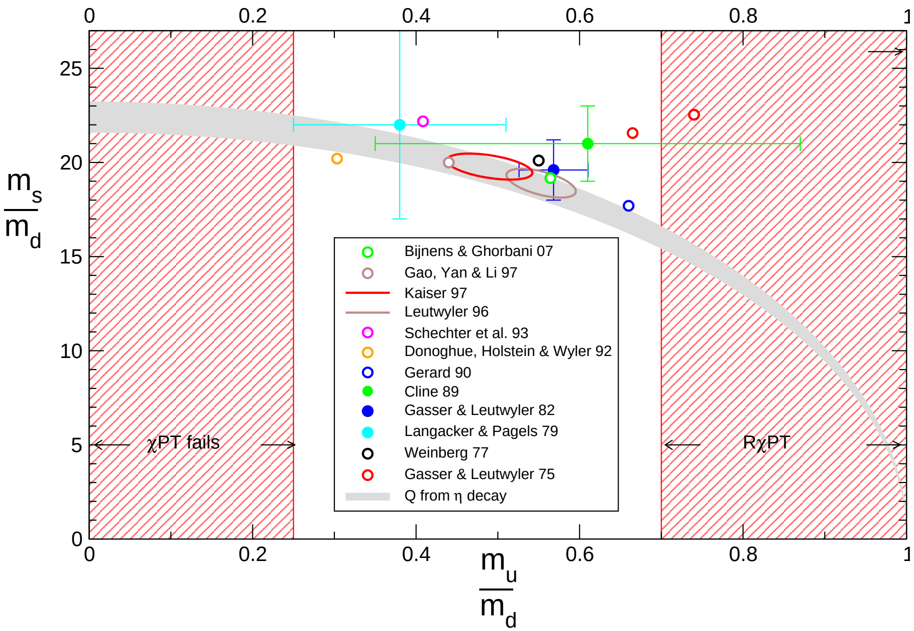
Figure taken from diploma work of Roland Kaiser (1997)

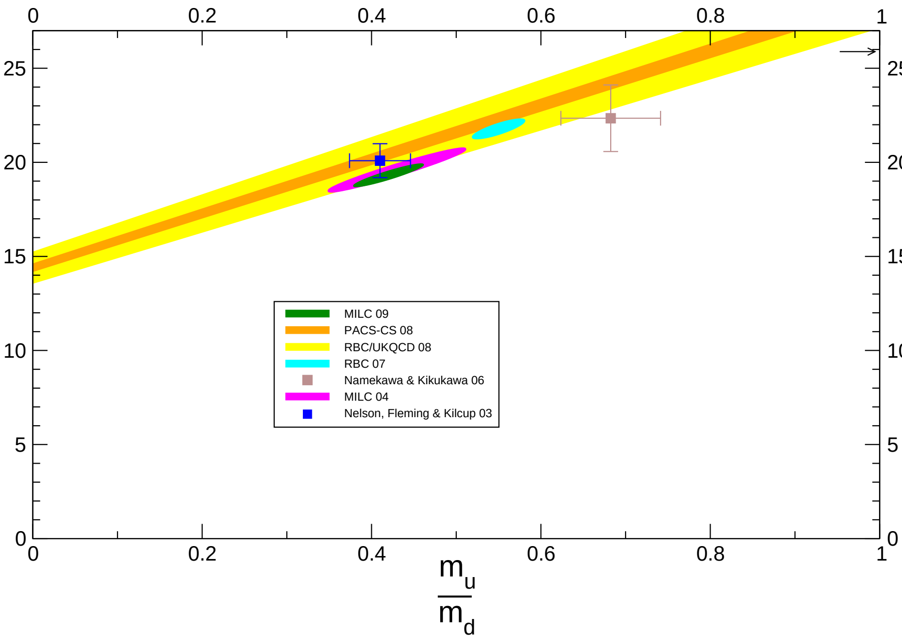
Tilted lines: value of $S = m_\mathrm{s}/m_\mathrm{ud}$, rectangle: experiment

Central value found in this determination: S = 26.6

Barely differs from leading order result: S = 25.9

Results for quark mass ratios

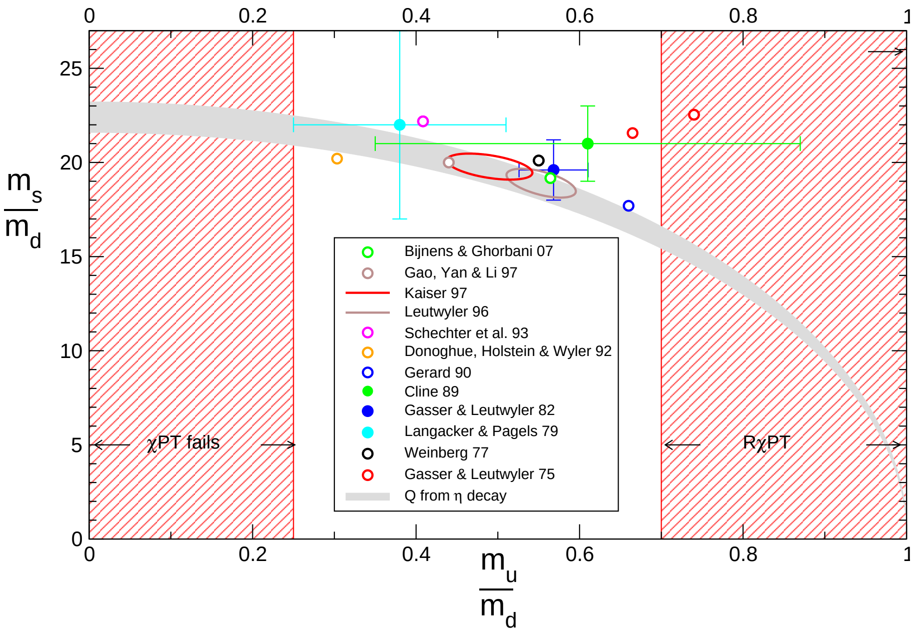




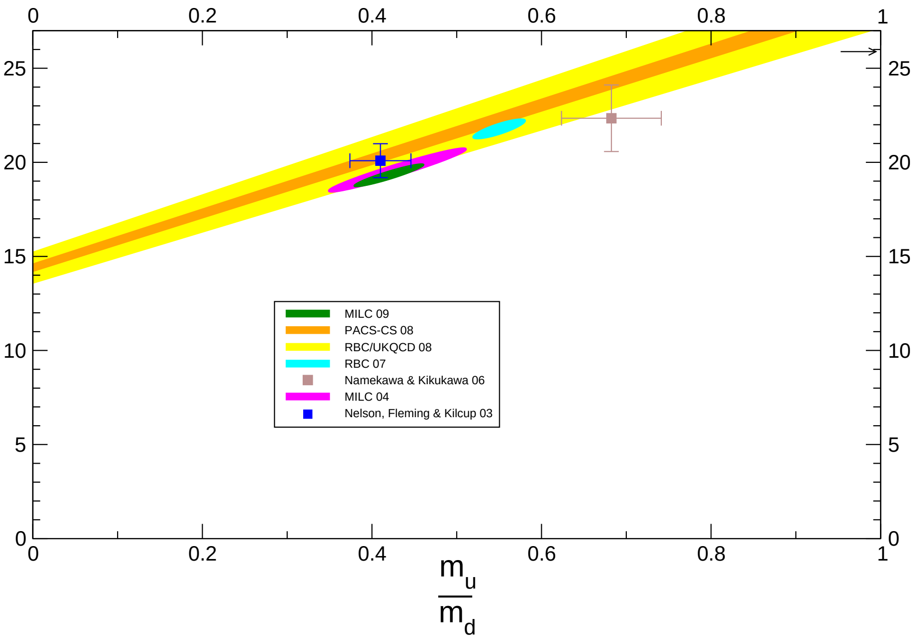
Phenomenology

Lattice

Results for quark mass ratios



Phenomenology

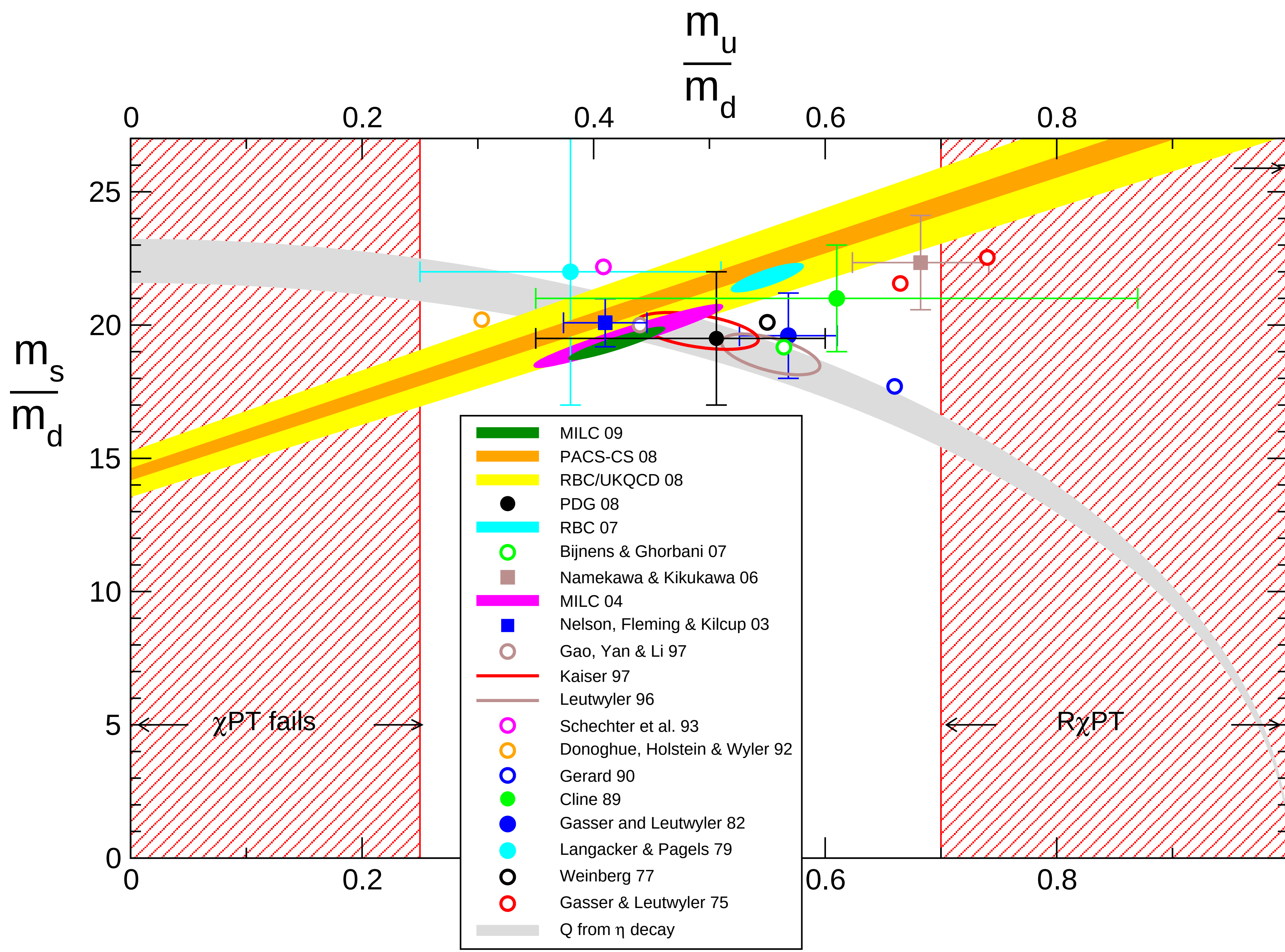


Lattice

None of the lattice results for m_u is consistent with the solution $m_u = 0$ of the strong CP problem

The MILC collaboration rules this solution out at 10 σ

Comparison



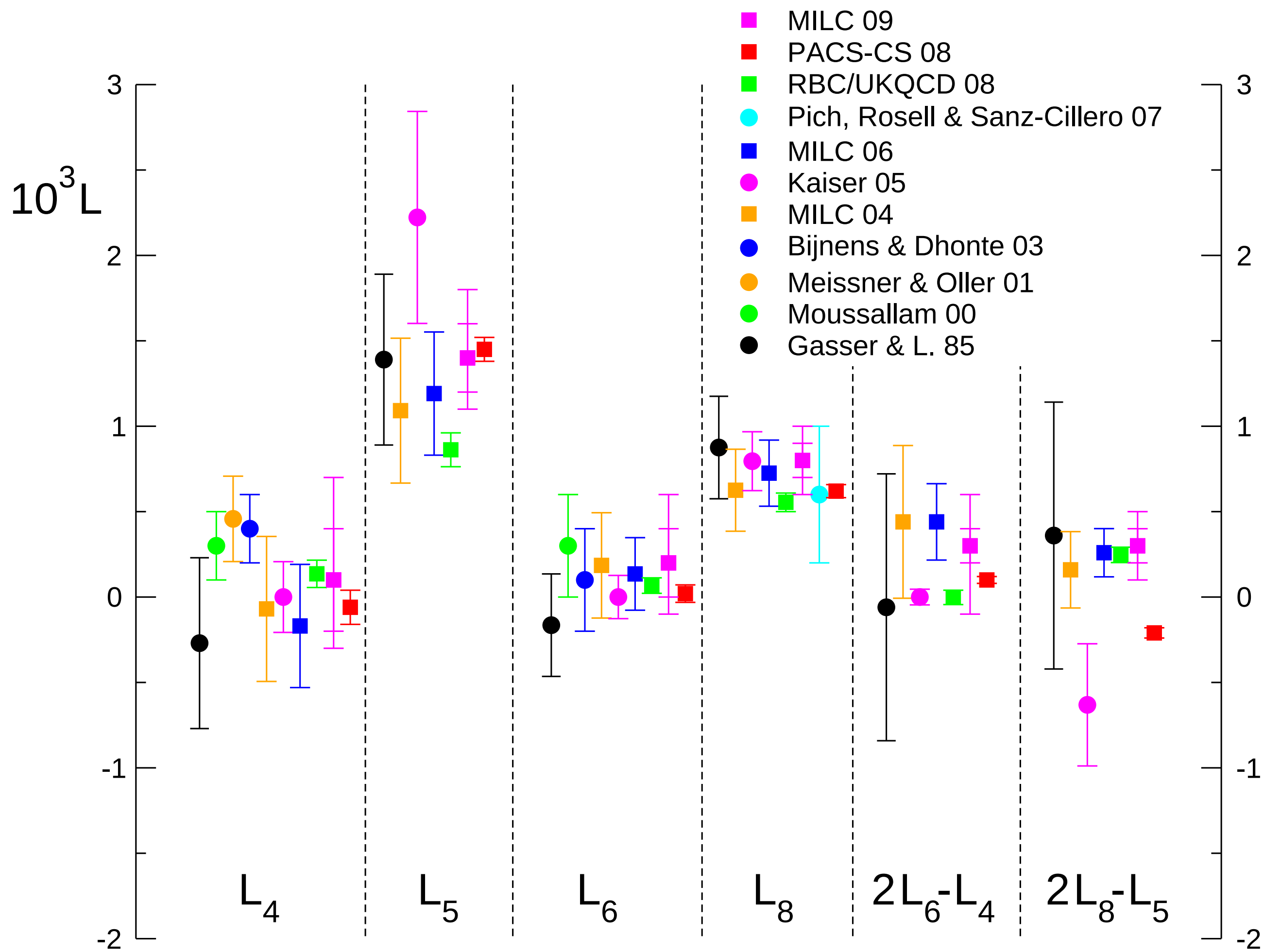
$\frac{m_u}{m_d}$
0
0.2
0.4
0.6
0.8
1
$\frac{m_s}{m_d}$
25
20
15
10
5
0
χPT fails
RχPT
MILC 09
PACS-CS 08
RBC/UKQCD 08
PDG 08
RBC 07
Bijnens & Ghorbani 07
Namekawa & Kikukawa 06
MILC 04
Nelson, Fleming & Kilcup 03
Gao, Yan & Li 97
Kaiser 97
Leutwyler 96
Schechter et al. 93
Donoghue, Holstein & Wyler 92
Gerard 90
Cline 89
Gasser and Leutwyler 82
Langacker & Pagels 79
Weinberg 77
Gasser & Leutwyler 75
Q from η decay

Effective coupling constants

- Apart from L_4, L_6, all of the SU(3)$_L \times$SU(3)$_R$ couplings of NLO can be determined from experiment Gasser + L. 1985
- L_4, L_6 are suppressed in large N_c limit, violate OZI rule

$\Rightarrow$ Expect L_4, L_6 to be small (at a running scale where large logarithms do not occur, such as $\mu = M_\eta$)

NLO couplings: L_4, L_5, L_6, L_8



Numerical values shown refer to running scale $\mu = M_\rho$

Lattice results for L_4, L_5, L_6, L_8 agree with phenomenology within errors

The large N_c suppression of L_4, L_6 is confirmed

At NLO, the position on the ellipse is determined by $2L_8 - L_5$

Conclusion

- $m_{\mathrm{u}} \neq 0$
  Nature solves the strong CP problem differently
- Lattice yields remarkably coherent and significant results for pion physics already now

$\Rightarrow$ SU(2)$\times$SU(2) χPT has become a precision tool

- Extension to kaon physics is making progress
  - M_K = 600 MeV is beyond reach of NLO χPT
  - Representations of many quantities of interest are available to NNLO of χPT $\Rightarrow$ talk by Hans Bijnens
  - Main problem at NNLO: the current knowledge of the LECs is rudimentary
  - NNLO formulae are needed in a form suitable for the analysis of lattice data

Conclusion ctd.

- m_s/m_d and m_s are now known to about 10 %
- m_u/m_d is known to about 20 %
- For the physical values of m_u, m_d, m_s, the leading order terms in the chiral perturbation series of M_π, M_K, F_π, F_K do represent a good approximation
- Lattice results indicate that the NLO contributions do dominate the corrections

$\Rightarrow$ χPT does appear to work

- Significant progress at the interface between lattice and effective field theory methods is ante portas