Spontaneous chiral symmetry breaking on the lattice

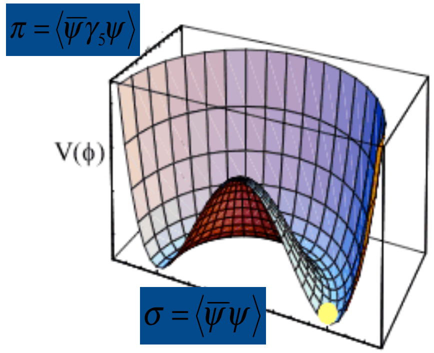
Shoji Hashimoto (KEK)

@ Chiral Dynamics 09 (Bern), Jul 6, 2009.

Chiral symmetry breaking

- QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \bar{\psi}(\not{D} + m)\psi$$



$$\langle \bar{\psi}\psi \rangle \neq 0$$

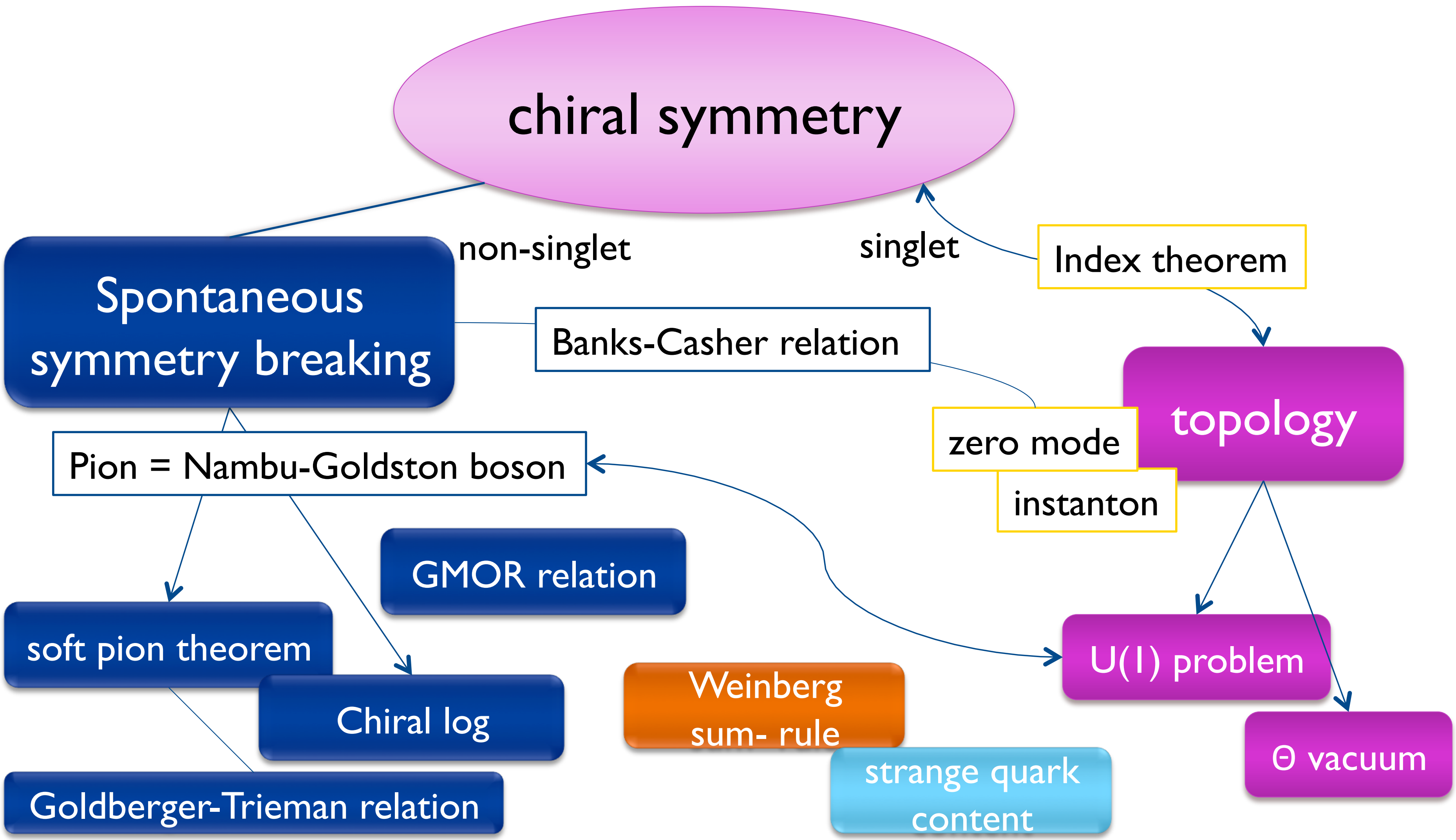
Spontaneous chiral symmetry breaking

- Chiral effective theory

$$\mathcal{L}_{\mathrm{ChPT}} = \frac{f^2}{16} \mathrm{Tr}\left[\partial_\mu U \partial_\mu U^\dagger\right] + \frac{m\Sigma}{2} \mathrm{Tr}\left[U + U^\dagger\right] + \ldots$$

  - GMOR relation, Goldberger-Treiman relation, Weinberg sum rule, soft pion theorem, chiral log, strange quark content, U(1) problem, θ vacuum, …

Chiral symmetry is *the* key



Chiral symmetry on the lattice

- Lattice QCD (with the Wilson-type fermions):

  Chiral symmetry is explicitly violated (Nielsen-Ninomiya, 1981)

  - Not clear how to discriminate between physics and artifacts
  - e.g. chiral condensate: additive renormalization exists. Hard to subtract the power divergence.

$$(\bar{\psi}\psi)^{cont} = Z_S(\bar{\psi}\psi)^{lat} + Z_{mix}\frac{1}{a^3}(1)^{lat}$$

- staggered fermion has chiral symmetry, but breaks flavor symmetry.
  - Might not be in the same universality class; needs careful continuum limit.
  - Needs the rooted determinant $(\det D)^{1/2}$

Chiral condensate $\langle \bar{q}q \rangle$

- Order parameter of the chiral symmetry
  - direct prove of SSB
- Not easy to calculate even with exact chiral symmetry
  - Power divergence persists except in the massless limit; makes sense only in the chiral limit

$$\langle \bar{q}q \rangle^{cont} = Z_S \langle \bar{q}q \rangle^{latt} + m_q \frac{c_1}{a^2}$$

  - Thermodynamical limit: No SSB at finite volume; vanishes unless measured in the infinite volume limit

Need some theoretical guidance; provided by ChPT

This talk

= an attempt to simulate the QCD vacuum on the lattice with *exact* chiral symmetry

- Overlap fermion (Neuberger, Narayanan, 1998)
  - Chiral and flavor symmetries are exact at finite lattice spacing *a*
  - Correctly reproduces the axial-anomaly (and the index theorem)
- Study of spontaneous chiral symmetry breaking
  - Precise calculation of chiral condensate
    - spectral density + topological susceptibility, GMOR, …
  - Testing the chiral effective theory beyond the tree level

Plan

1. Dirac spectrum and chiral symmetry breaking
   - ε-regime and p-regime
   - beyond the leading order
2. Lattice calculation of the Dirac spectrum
   - Setup (not in detail)
   - Results for the spectral density
3. Other consequences of SSB
   - Topological susceptibility
   - Vacuum polarization functions
   - Convergence of the chiral expansion (m_π, f_π)
4. Conclusions

1. Dirac operator spectrum and chiral symmetry breaking

Banks-Casher relation

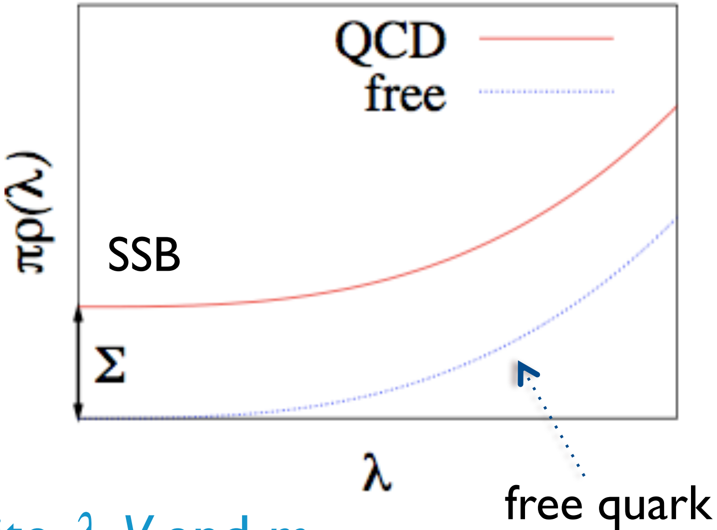
- Bose-Einstein condensation in the QCD vacuum
  - Spectral density of the Dirac operator carries the info of the spontaneous symmetry breaking.

$$\rho(\lambda) = \frac{1}{V}\sum_k \langle \delta(\lambda - \lambda_k) \rangle,$$

$$\langle \bar{q}q \rangle = \frac{1}{V}\sum_k \left\langle \frac{1}{m + i\lambda_k} \right\rangle$$

Banks-Casher (1980)

$$\lim_{m\to 0}\lim_{V\to\infty} \rho(\lambda = 0) = \frac{\Sigma}{\pi}$$

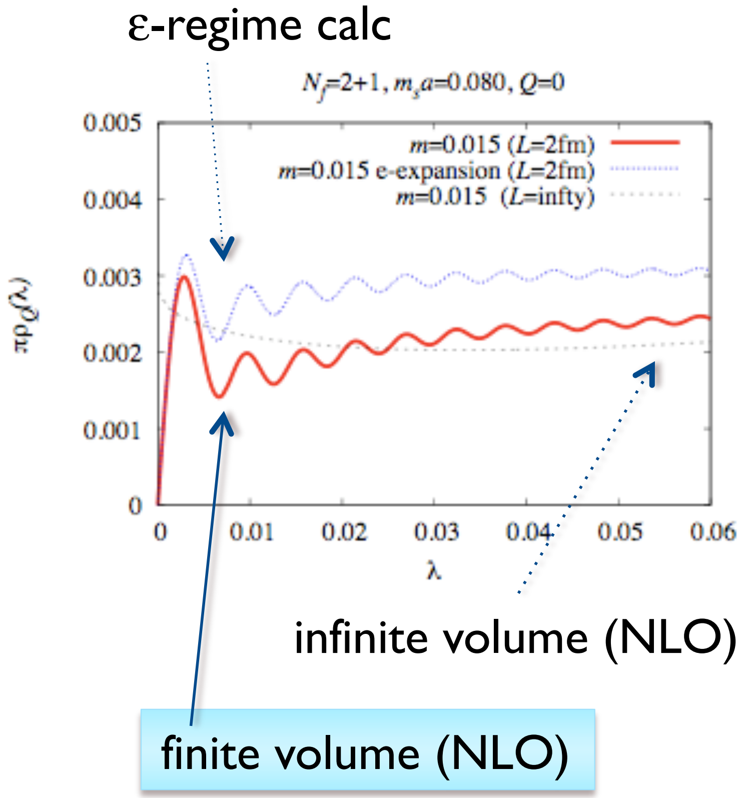


  - Also possible to study the relation at finite λ, V and m.
    - finite λ at NLO(p): Smilga-Stern (1993).
    - finite λ and m at NLO(p): Osborn-Toublan-Verbaarschot (1999).
    - finite V and small λ and m at NLO(ε): Damgaard-Nishigaki (1998).

SSB on the lattice

- Symmetry breaking occurs only in the infinite volume.
  - Need to study the finite volume scaling for a rigorous test.
  - Still possible to study on the finite volume lattice with the help of ChPT.
- Beyond the leading order
  - New formula valid in both the p- ($m_\pi L > 1$) and the ε-regime ($m_\pi L < 1$), and in between.
    - Damgaard-Fukaya, JHEP 0901, 052 (2009).
    - zero mode integral done even in the p-regime

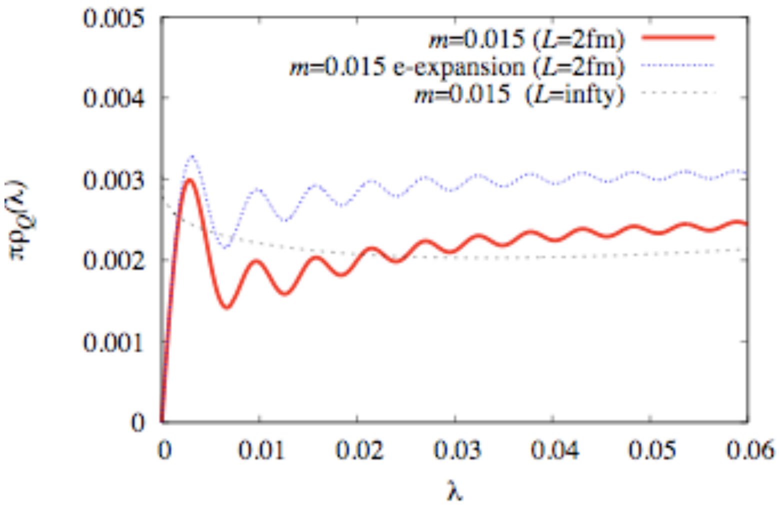
Damgaard-Fukaya (2009)
ex) m_π~300 MeV, L~2fm



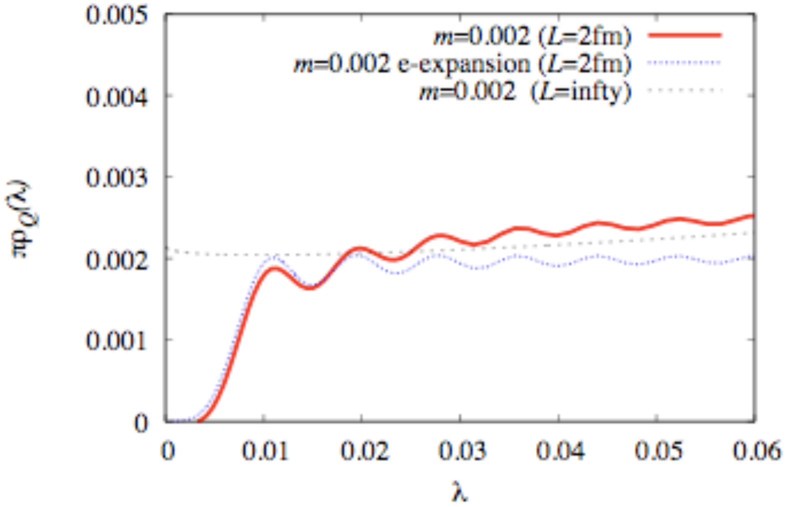
Expectation

- Once we could calculate the spectral density on a finite volume lattice (L ~ 2 fm) …

p-regime: m_π ~ 300 MeV



ε-regime: m_π ~ 100 MeV



  - Height determines Σ at the NLO accuracy
  - Shape is related to the NLO effects ~ $1/F^2$

2. Lattice setup (not in detail)

JLQCD+TWQCD collaborations

- JLQCD
  - SH, H. Ikeda, T. Kaneko, H. Matsufuru, J. Noaki, N. Yamada (KEK)
  - H. Fukaya (Nagoya)
  - T. Onogi, E. Shintani (Osaka)
  - H. Ohki (Kyoto)
  - S. Aoki, N. Ishizuka, K. Kanaya, Y. Kuramashi, K. Takeda, Y. Taniguchi, A. Ukawa, T. Yoshie (Tsukuba)
  - K. Ishikawa, M. Okawa (Hiroshima)
- TWQCD
  - T.W. Chiu, T.H. Hsieh, K. Ogawa (National Taiwan Univ)
- Machines at KEK (since 2006)
  - SR11000 (2.15 Tflops)
  - BlueGene/L (10 racks, 57.3 Tflops)

Project: dynamical overlap fermions

First large scale simulation with *exact* chiral symmetry

Theoretical interest	Phenomenological interest
• Dirac operator spectrum: Banks-Casher relation, chiral RMT • Chiral symmetry breaking: chiral condensate and related • Topology: θ-vacuum, topological susceptibility	• Controlled chiral extrapolation with the *continuum* ChPT • Physics applications: B_K, form factors, etc. • Sum rules, OPE • Flavor-singlet physics

Publications from the project

Not including conference proceedings

1. Fukaya et al. "*Lattice gauge action suppressing near-zero modes*," Phys. Rev. D, 094505 (2006).
2. Fukaya et al. "*Two-flavor QCD simulation in the ε-regime…*," Phys. Rev. Lett 98, 172001 (2007).
3. Fukaya et al. "*Two-flavor lattice QCD in the ε-regime…*," Phys. Rev. D76, 054503 (2007).
4. Aoki, Fukaya, SH, Onogi, "*Finite volume QCD at fixed topological charge*," Phys. Rev. D76, 054508 (2007).
5. Aoki et al., "*Topological susceptibility in two-flavor QCD…*," Phys. Lett. B665, 294 (2008).
6. Fukaya et al., "*Lattice study of meson correlators in the ε-regime…*," Phys. Rev. D77, 074503 (2008).
7. Aoki et al. "*B_K with two flavors of dynamical overlap fermions*," Phys. Rev. D77, 094503 (2008).
8. Aoki et al. "*Two-flavor QCD simulation with exact chiral symmetry*," Phys. Rev. D **78**, 014508 (2008); arXiv: 0803.3197 [hep-lat].
9. Noaki et al. "*Convergence of the chiral expansion…*," Phys. Rev. Lett. 101, 202004 (2008); arXiv: 0806.0894 [hep-lat].
10. Shintani et al. "*S-parameter and pseudo NG boson mass…*," Phys. Rev. Lett. 101, 242001 (2008); arXiv: 0806.4222 [hep-lat].
11. Ohki et al., "*Nucleon sigma term and strange quark content…*," Phys. Rev. D **78**, 054502 (2008); arXiv: 0806.4744 [hep-lat] .
12. Shintani et al., "*Lattice study of the vacuum plarization functions and …*," Phys. Rev. D **79**, 074510 (2009); arXiv:0807.0556 [hep-lat].
13. S. Aoki et al., "*Pion form factors from two-flavor lattice QCD with exact chira symmetry*," arXiv:0905.2465 [hep-lat].

Overlap fermion

- Neuberger-Narayanan (1998)
  - constructed with the Wilson fermion as a kernel

$$D = \frac{1}{a}\left[1 + \frac{X}{\sqrt{X^\dagger X}}\right],\ X = aD_W - 1$$
$$= \frac{1}{a}\left[1 + \gamma_5\,\mathrm{sgn}(aH_W)\right],\ aH_W = \gamma_5(aD_W - 1)$$

$$D\gamma_5 + \gamma_5 D = aD\gamma_5 D$$

- Exact chiral symmetry through the Ginsparg-Wilson relation.
  - Continuum-like Ward-Takahashi identities hold.
  - Index theorem (relation to topology) satisfied.
  - Topology change is costly; large-scale simulation is feasible only at fixed topology
    - induces O(1/V) effects in general
    - No problem for the spectral function analysis

Parameters

N_f = 2 runs

- β=2.30 (Iwasaki), a=0.12 fm, 16^3x32
- 6 sea quark masses covering $m_s/6 \sim m_s$
- Q=0 sector only, except for Q=-2, -4 runs at m_q=0.050
- ε-regime run at m=0.002 ($m_q \sim$ 3 MeV), β=2.30

N_f = 2+1 runs

- β=2.30 (Iwasaki), a=0.11 fm, 16^3x48
- 5 ud quark masses, covering $m_s/6 \sim m_s$
  - x 2 s quark masses
- Q=0 sector only, except for Q=1 at m_{ud}=0.015
- Larger volume lattice 24^3x48 running at m_{ud}=0.015, 0.025.
- ε-regime run at m=0.002 ($m_q \sim$ 3 MeV)

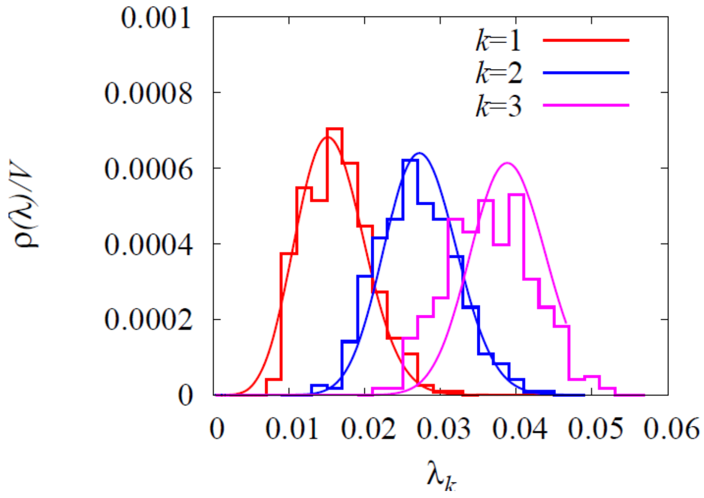
Early analysis (~2007)

- N_f=2 in the ε-regime
  - low-mode distribution compared with the Random Matrix Theory (RMT) to extract Σ.

    $\Sigma(2\text{ GeV}) = [\,251(7)(11)\text{ MeV}]^3$

  - Valid for small enough $\lambda \sim 1/\Sigma V$
- Limitations
  - Controlled finite volume effects?
  - p-regime lattice not useful
  - Not possible to extend RMT to NLO

JLQCD, Phys. Rev. Lett 98, 172001 (2007)

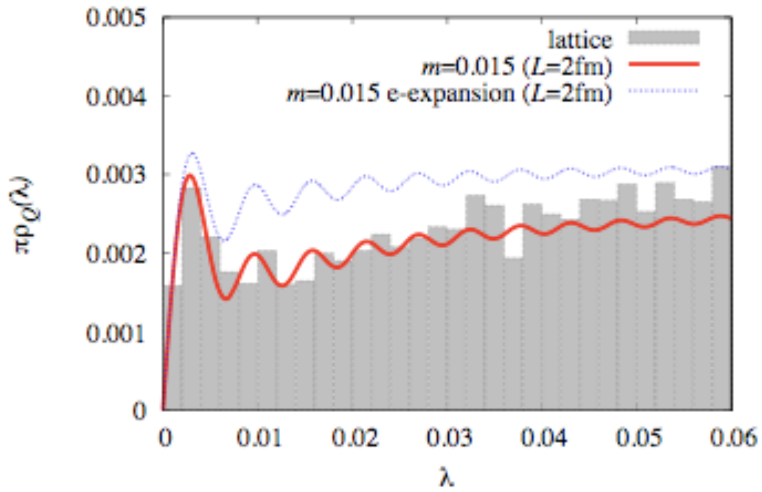


Can overcome with the new ChPT formulae.

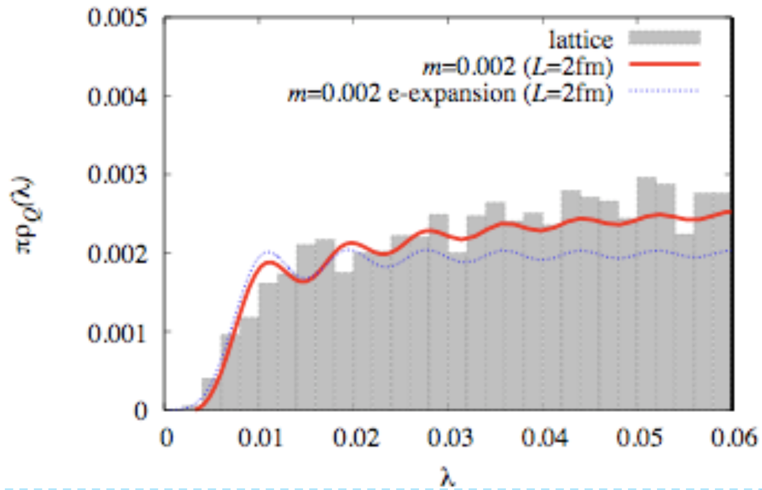
New analysis (2009)

- Direct use of the spectral function
  - 2+1-flavor data
  - Uses both the p-regime and ε-regime lattices.
  - Fit the whole shape against the ChPT formula.
- Comparison
  - NLO formula reproduces the lattice data precisely.
  - The previous ε-regime formula was useful only for the 1st eigenvalue in the p-regime.

p-regime



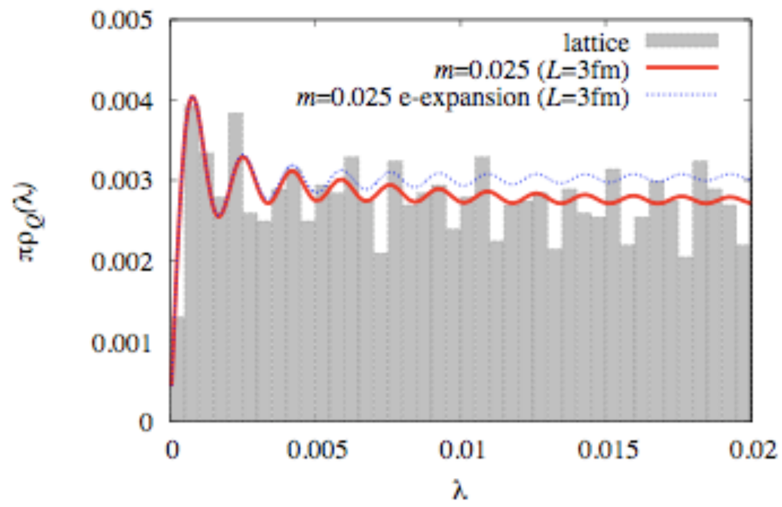
ε-regime



New analysis (2009)

- Finite volume effect
  - Checked with a larger volume data on a 24^3x48 lattice (L ~ 2.6 fm)
  - Can be fitted with the same set of parameters as in the 16^3x48 analysis.
    - $\rho(\lambda)$ slightly going down after the first peak = pion-loop effect in the p-regime.
  - Finite volume effect well under control.

24^3x48

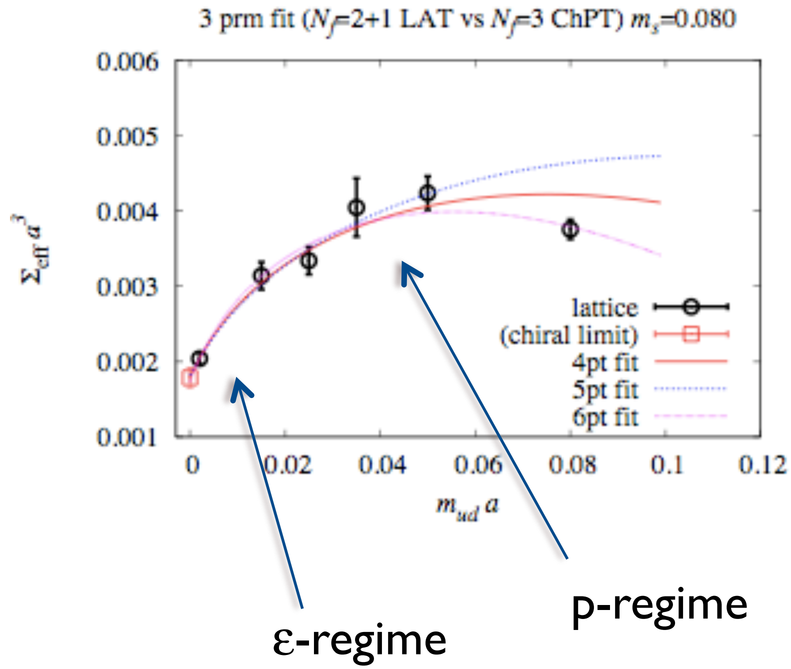


Chiral extrapolation

- Lattice data at 6 values of m_{ud} including the ε-regime
  - Massless limit of up-down, while keeping strange quark mass at its physical value.
  - Chiral log

$$\Sigma(m_{ud},m_s)=\Sigma(0,m_s)\times\left[1-\frac{3M_\pi^2}{32\pi^2F^2}\ln\frac{M_\pi^2}{\mu^2}+\frac{32L_6M_\pi^2}{F^2}\right]$$

well reproduced by the lattice data.
    - Fit is done with N_f=2 and with N_f=2+1 formulae.
  - Determination of Σ and *F*, L_6



JLQCD (2009); preliminary

$$\Sigma^{\overline{MS}}(0,m_s;2\ \mathrm{GeV})=\left[243(4)(^{+16}_{-0})\ \mathrm{MeV}\right]^3$$

$$F\sim 80(2)(5)\ \mathrm{MeV}$$

$$L_6^r=-0.00012(9)(10)$$

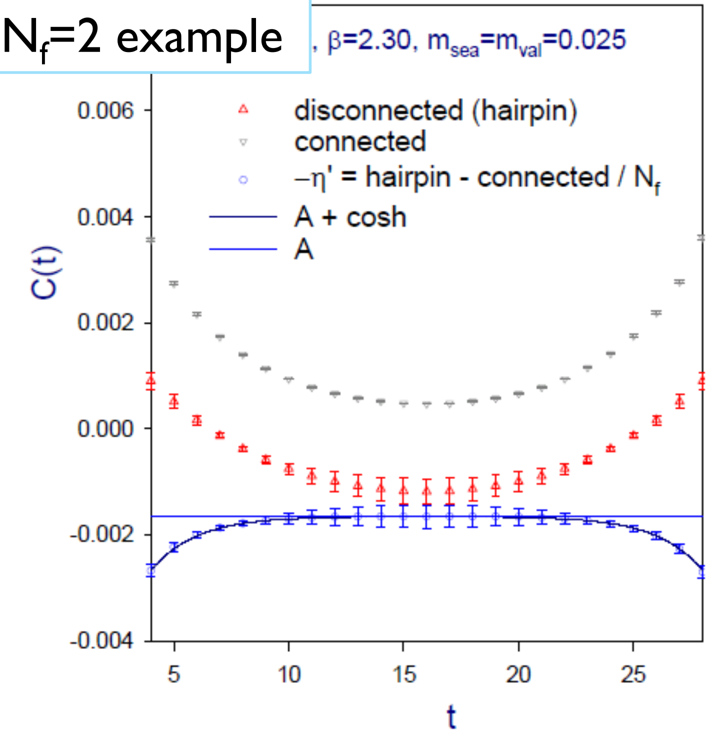
3. Other consequences of SSB

Topological susceptibility $\chi_t = \langle Q^2 \rangle / V$

- Correlation of the topological charge density at fixed Q
  - ~ constant proportional to χ_t

$$\lim_{x\to\infty}\langle mP(x)mP(0)\rangle_Q = -\frac{1}{V}\left(\chi_t - \frac{Q^2}{V} + \ldots\right) + O(e^{-m_{\eta'}x})$$

N_f=2 example

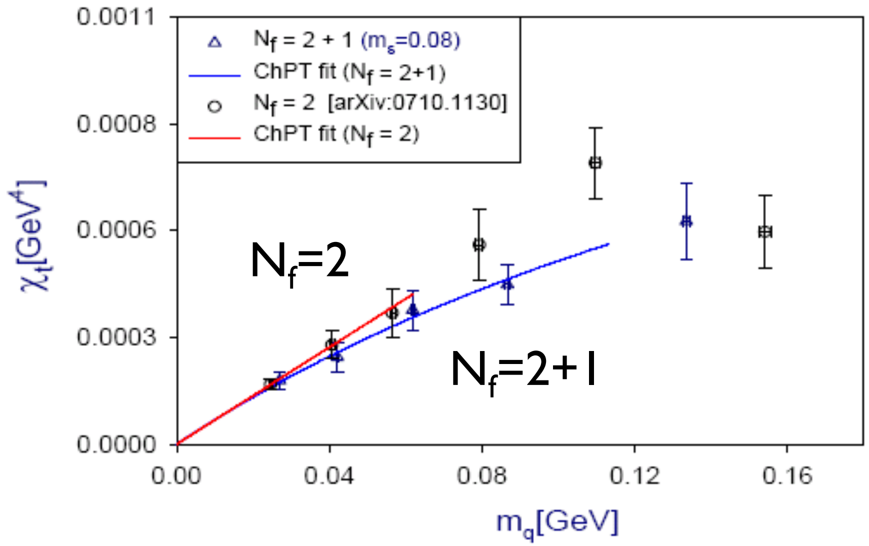


JLQCD, Phys. Lett. B665, 294 (2008)

(negative) constant correlation of the local topological charges clearly seen.

- Results from other topological sectors are consistent.
- Higher order correction (~1/V^2) also estimated using 4-point corr.

Sea quark mass dependence



JLQCD (2009): N_f=2 and 2+1

Disconnected loops constructed from low modes (saturation confirmed).

- The effect of SSB = vanishing towards the chiral limit

Crewther (1977), Leutwyler-Smilga (1992)

$$\chi_t = m\Sigma / N_f, \quad \text{or} \quad \chi_t = \frac{\Sigma}{m_u^{-1} + m_d^{-1} + m_s^{-1}}$$

- Fit with ChPT expectation
  - N_f=2:
    Σ = [242(5)(10) MeV]3
  - N_f=2+1:
    Σ = [247(3)(2) MeV]3

Vacuum polarization functions

- Vector and axial correlators in the momentum space.

$$\langle J_\mu J_\nu \rangle = \left(g_{\mu\nu} q^2 - q_\mu q_\nu\right)\Pi_J^{(1)}(Q^2) - q_\mu q_\nu \Pi_J^{(0)}(Q^2)$$
$$= \int_0^\infty \frac{ds}{s - q^2 + i\varepsilon}\left[\left(g_{\mu\nu} s^2 - s_\mu s_\nu\right)\mathrm{Im}\,\Pi_J^{(1)}(s) - s_\mu s_\nu\,\mathrm{Im}\,\Pi_J^{(0)}(s)\right]$$

  - Directly calculable on the lattice for space-like momenta
- Weinberg sum rules:

$$f_\pi^2 = -\lim_{Q^2\to 0} Q^2\left[\Pi_V^{(1+0)}(Q^2) - \Pi_A^{(1+0)}(Q^2)\right],$$
$$S = -\lim_{Q^2\to 0}\frac{\partial}{\partial Q^2} Q^2\left[\Pi_V^{(1+0)}(Q^2) - \Pi_A^{(1+0)}(Q^2)\right] \quad \text{or } L_{10}$$

  - Vanishes when V=A; another probe of SSB
  - S is relevant for the precision EW test of new strong dynamics.

Pion electromagnetic mass splitting

- Das-Guralnik-Mathur-Low-Young sum rule (1967)

$$\Delta m_\pi^2 = -\frac{3\alpha_{\rm EM}}{4\pi f_\pi^2}\int_0^\infty dQ^2\ Q^2\left[\Pi_V^{(1+0)}(Q^2)-\Pi_A^{(1+0)}(Q^2)\right]$$

  - Valid in the chiral limit (soft pion theorem)
  - Gives dominant contribution to the $\pi^\pm$-π^0 splitting.
  - Related to the pseudo-NG boson mass in the context of new strong dynamics.
- Exact chiral symmetry is essential.
  - The quantity of interest is obtained after huge cancellation between V and A.

Lattice results

- Can be fitted with
  - ChPT in the low q^2 region

$$\Pi^{(1)}_{V-A}(q^2) = -\frac{f_\pi^2}{q^2} - 8L_{10}^r(\mu) - \frac{1}{24\pi^2}\left[\ln\frac{m_\pi^2}{\mu^2} + \frac{1}{3} - H(x)\right]$$

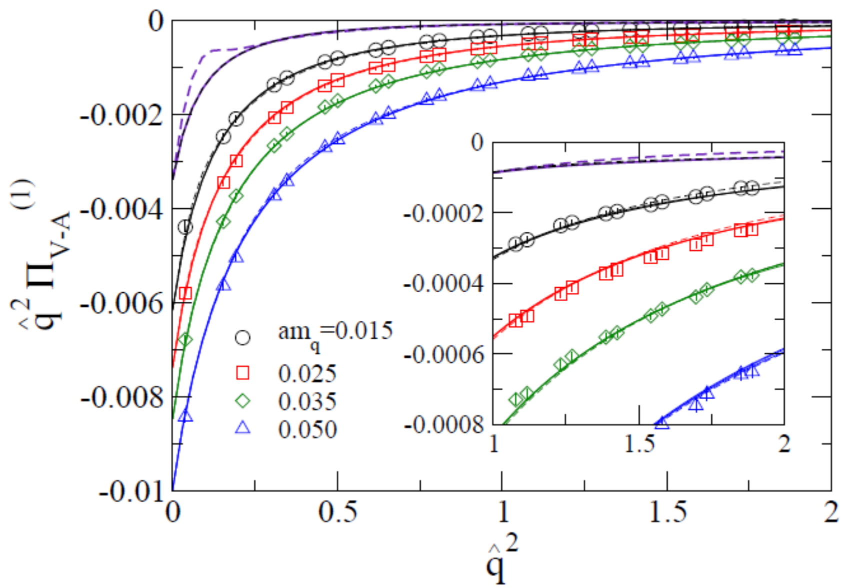
  L$_{10}$ is extracted.

$$L_{10}^r(m_\rho) = -5.2(2)(^{+0}_{-3})(^{+5}_{-0})\times 10^{-3}$$

  - OPE in the high q^2 region. In the massless limit, $1/Q^6$ is the leading.
  - Summing up the two regions, Δm_π^2 is obtained.

JLQCD, Phys. Rev. Lett. 101, 242001 (2008)

N_f=2



$$\Delta m_\pi^2 = 993(12)(^{+\ 0}_{-135})(149)\ \text{MeV}^2$$

Exp: Δm_π^2= 1261.2 MeV^2

Pion mass & decay constant

Phys. Rev. Lett. 101, 202004 (2008)

Precise test of GMOR

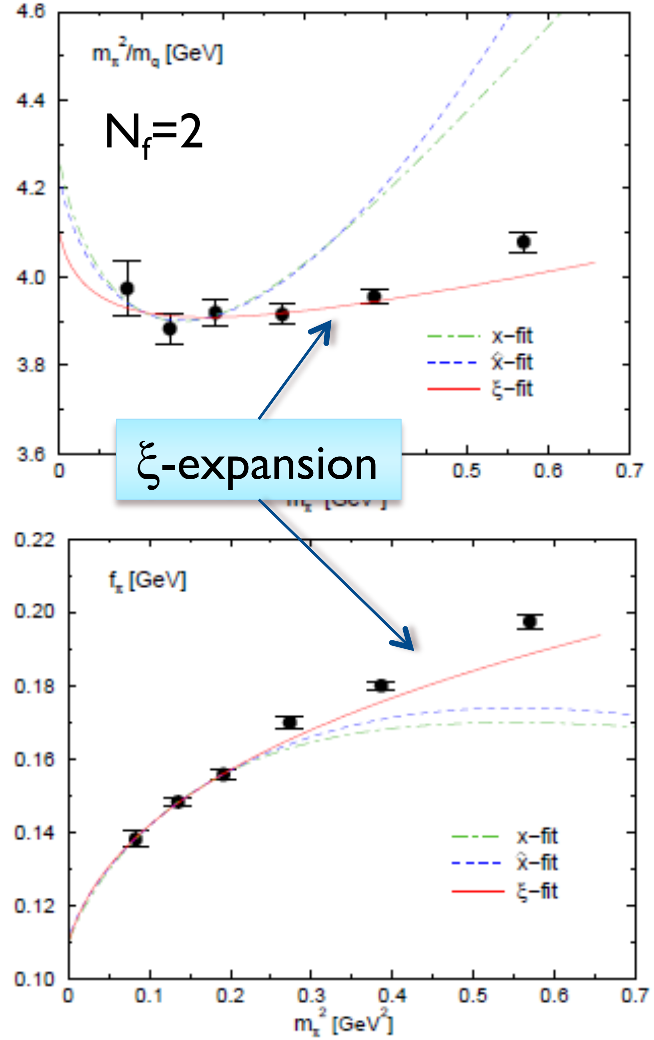
- Chiral expansion
  - The region of convergence is not known a priori.
  - Test on the lattice with exact chiral symmetry

$$\frac{m_\pi^2}{m_q} = 2B\left[1 + x\ln x + c_3 x + O(x^2)\right],$$

$$f_\pi = f\left[1 - 2x\ln x + c_4 x + O(x^2)\right].$$

  - Expand in either

$$x \equiv \frac{m^2}{(4\pi f)^2},\ \hat{x} \equiv \frac{m_\pi^2}{(4\pi f)^2},\ \xi \equiv \frac{m_\pi^2}{(4\pi f_\pi)^2}$$



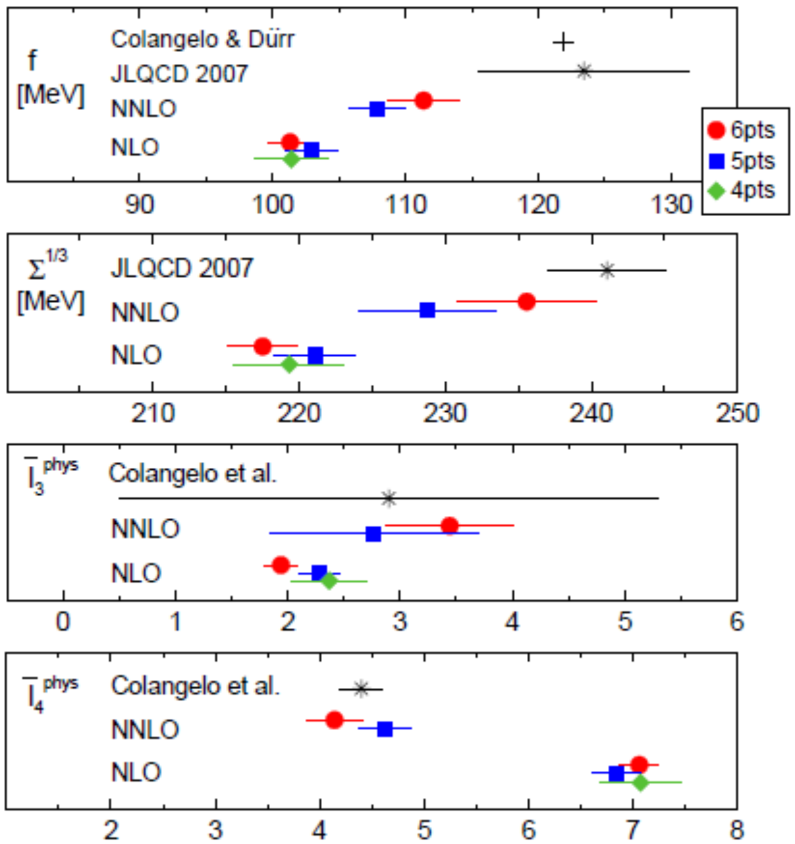
Two-loop analysis

- Analysis including NNLO
  - With the ξ-expansion

$$m_\pi^2/m_q = 2B\Big[1 + \xi\ln\xi + \tfrac{7}{2}(\xi\ln\xi)^2 + \Big(\tfrac{c_4}{2f} - \tfrac{4}{3}(\tilde{l}^{\text{ phys}} + 16)\Big)\xi^2\ln\xi\Big] + c_3\,\xi(1 - 9\xi\ln\xi) + \alpha\,\xi^2,$$

$$f_\pi = f\Big[1 - 2\xi\ln\xi + 5(\xi\ln\xi)^2 + \tfrac{3}{2}(\tilde{l}^{\text{ phys}} + \tfrac{53}{2})\xi^2\ln\xi\Big] + c_4\,\xi(1 - 10\xi\ln\xi) + \beta\,\xi^2.$$

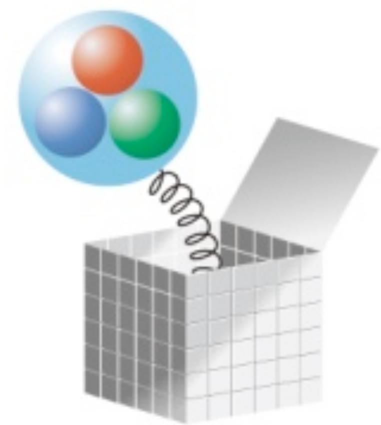
  - For reliable extraction of the low energy constants, the NNLO terms are mandatory.



Conclusions

- *The* spontaneous chiral symmetry breaking of QCD is confirmed by simulations with exact chiral symmetry.
  - Beyond LO; finite volume, chiral limit well under control.
  - Other consequences: topological susceptibility, Weinberg sum rules, GMOR, …
  - Overlap simulations open up new possibilities to extract physics from lattice.

At last, lattice QCD has followed up the various theoretical conjectures for strong interaction in 1960s and 70s. But now from first-principles.

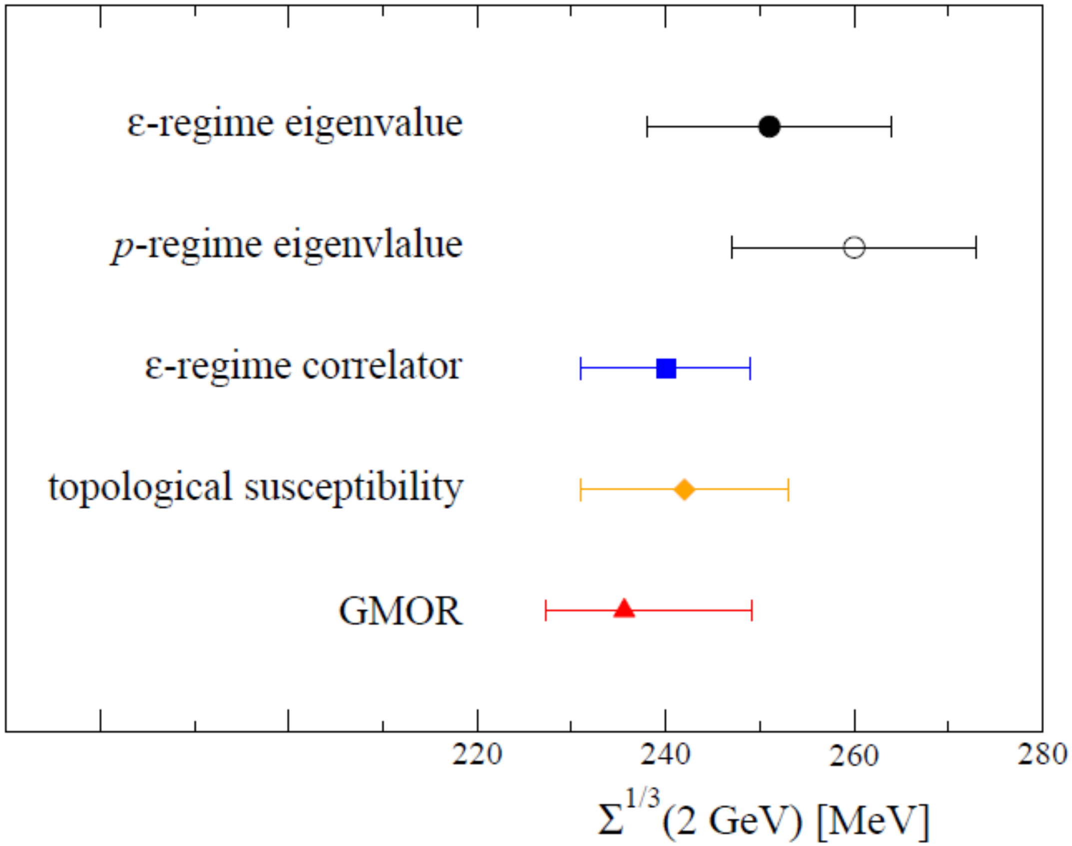


Thank you for your attention!

Backup slides

Two-flavor condensate

$\Sigma^{1/3}(2\,\mathrm{GeV})$



Phys. Rev. Lett 98, 172001 (2007)

Phys. Rev. D76, 054503 (2007)

Phys. Rev. D77, 074503 (2008)

Phys. Lett. B665, 294 (2008)

Phys. Rev. Lett. 101, 202004 (2008)

In good agreement

Pion form factors

- Another testing ground of ChPT
  - Vector and scalar

$$\langle \pi(p')|V_\mu|\pi(p)\rangle = i(p_\mu + p_\mu')F_V(q^2),$$

$$\langle \pi(p')|S|\pi(p)\rangle = F_S(q^2),\ q_\mu \equiv p_\mu' - p_\mu$$

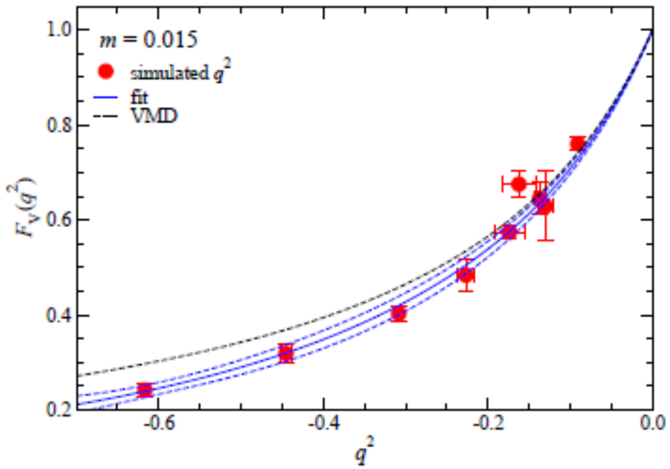
  - Charge and scalar radius

$$F_V(q^2) = 1 + \frac{1}{6}\langle r^2\rangle_V^\pi q^2 + O(q^4),$$

$$F_S(q^2) = F_S(0)\left[1 + \frac{1}{6}\langle r^2\rangle_S^\pi q^2 + O(q^4)\right],$$

  - Calculation using the all-to-all technique.

Vector form factor



q^2 dependence well described by a vector meson pole + corrections.

$$F_\pi(q^2) = \frac{1}{1 - q^2/m_V^2} + c_1 q^2 + \ldots$$

Chiral extrapolation

- Fit with NNLO ChPT
  - Data do not show clear evidence of the chiral log. But, it is expected to show up even smaller pion masses.
  - NNLO contribution is significant; necessary to reproduce the phenomenological values.