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Hadronic atoms

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Chiral Dynamics 2009, Bern, 7 July 2009

Plan

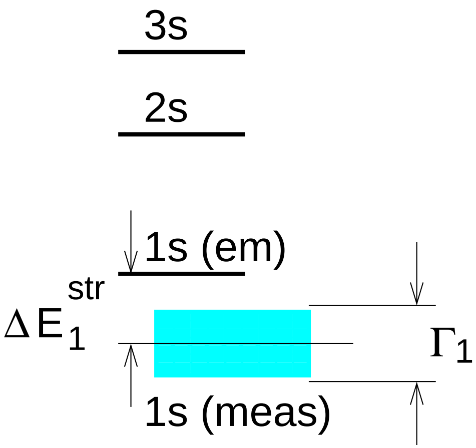
- Introduction
- Physics background
- Kaonic hydrogen
- EFT approach to kaonic deuterium: going beyond the static approximation
- Conclusions

An example: $\pi^+\pi^-$-atom

- Almost Coulombic bound state, decays predominately into $\pi^0\pi^0$
- $\langle \mathbf{p}^2\rangle^{1/2} = \alpha M_\pi/2 \simeq 0.5$ MeV $\ll M_\pi$
- $r_B = 2/\alpha m_\pi \simeq 387$ fm $\gg R_{\text{str}}$
- $E_B = M_\pi \alpha^2/4 + O(\alpha^3) \simeq 2$ keV $\ll M_\pi$

Measurable characteristics:

- Energy shift $\Delta E^{\text{str}} \ll E_B$
- Decay width $\Gamma \ll E_B$



DGBT formula

S. Deser, M.L. Goldberger, K. Baumann and W.E. Thirring, Phys. Rev. 96 (1954) 774

$$\Delta E^{\rm str}-\frac{i}{2}\,\Gamma=-2\alpha^3\mu_c^2\,\mathcal{A}+O(\alpha^4)$$

Real and imaginary parts of the threshold amplitude:

$$\begin{aligned}\text{Re}\,\mathcal{A} &\sim 2a_0+a_2+\cdots\\ \text{Im}\,\mathcal{A} &\sim p^\star(a_0-a_2)^2+\cdots\end{aligned}$$

$p^\star$ is the relative momentum of the $\pi^0\pi^0$ pair

Due to a huge difference in the strong and atomic scales, the atomic spectrum is expressed solely in terms of the effective-range expansion parameters of the $\pi\pi$ scattering amplitude

Physics background: $\pi^+\pi^-$

- Extract the combination $|a_0 - a_2|$ of the scattering lengths from the measured lifetime of the $\pi^+\pi^-$ atom in the ground state (DIRAC at CERN → Yazkov)
- Compare with the theoretical prediction

$$a_0 = 0.220 \pm 0.005\,, \qquad a_2 = -0.0444 \pm 0.0010$$
$$a_0 - a_2 = 0.265 \pm 0.004$$

G. Colangelo, J. Gasser and H. Leutwyler, NPB 603 (2001) 125

- Test large/small condensate scenario in QCD
- Alternative methods:

  Cusps in $K \to 3\pi$ decays (→ Giudici)
  K_{e4} decays (→ Bloch-Devaux)

Physics background: πK

- Allows to extract the combination $|a_{1/2} - a_{3/2}|$ of the πK scattering lengths from the lifetime measurement (DIRAC at CERN $\rightarrow$ Yazkov, Benelli)

- Tests the convergence of the chiral expansion in the meson sector in $SU(3) \times SU(3)$ ChPT

  V. Bernard, N. Kaiser and U.-G. Meißner, NPB 357 (1991) 129

  J. Bijnens, P. Dhonte and P. Talavera, JHEP 0405 (2004) 036

  J. Schweizer, PLB 625 (2005) 217

- Potentially, the measurement of the energy shift of the πK atom provides an opportunity to test the large/small condensate scenario in QCD with three flavors

  $\hookrightarrow$ Flavor-dependence of the quark condensate

Physics background: $\pi^- p$ and $\pi^- d$

- High-precision measurement of the πN scattering lengths (Pionic Hydrogen collaboration at PSI)

  Experimental uncertainty: energy shift 0.2 %, width 2%

  Theoretical uncertainty: isospin-breaking effects ($\rightarrow$ Hoferichter)

- πNN coupling constant through GMO sum rule
- Pion-nucleon σ-term through dispersion relations + ChPT
- Strangeness content of the nucleon

Physics background: K^-p and K^-d

- Extracting the antikaon-nucleon scattering lengths a_0, a_1 from the *combined* analysis of the kaonic hydrogen and kaonic deuterium data (DEAR/SIDDHARTA at DAFNE)

  Since scattering lengths are strongly absorptive, the measurement of the kaonic hydrogen alone does not suffice

- Confronting experimental result with the coupled-channel unitarized baryon ChPT in the $S = -1$ sector
- Solving the problem of incompatibility of scattering data with DEAR result
- Restricting the sub-threshold energy dependence of the K^-p amplitude

$\longrightarrow$ implications for the interactions of K^- with medium

Experimental status: kaonic hydrogen

$$\begin{aligned} \Delta E_{1s}^{\text{str}} &= 193 \pm 37 \text{ (stat)} \pm 6 \text{ (syst) MeV} \\ \Gamma_{1s} &= 249 \pm 111 \text{ (stat)} \pm 30 \text{ (syst) MeV} \end{aligned}$$

DEAR at DAFNE

U.-G. Meißner, U. Raha and AR, EPJC 35 (2004) 349

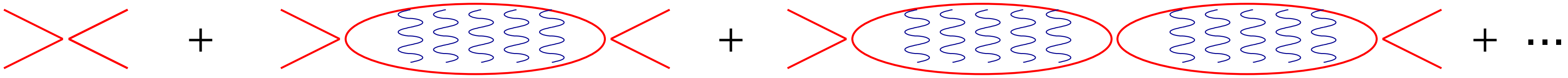
$$\Delta E_{1s} - i\,\frac{\Gamma_{1s}}{2} = -2\alpha^3\mu^2 a_p \Big(1 - \underbrace{2\mu\alpha(\ln\alpha - 1)a_p}_{\text{Coulomb}}\Big) + \cdots$$

$\hookrightarrow$ Precise determination of a_p:

$$a_p = \frac{\frac{1}{2}\,(a_0 + a_1) + q_0 a_0 a_1}{1 + \frac{q_0}{2}\,(a_0 + a_1)}\,,$$

$q_0 =$ threshold momentum of $\bar K^0 n$

Summing up Coulomb bubbles



$$1-\underbrace{2\alpha(\ln\alpha-1)}_{\simeq -0.09}\,\mu a_p \to \left(1+2\alpha(\ln\alpha-1)\,\mu a_p\right)^{-1}$$

agrees with: A. Cieply and J. Smejkal, EPJA 34 (2007) 237, potential model

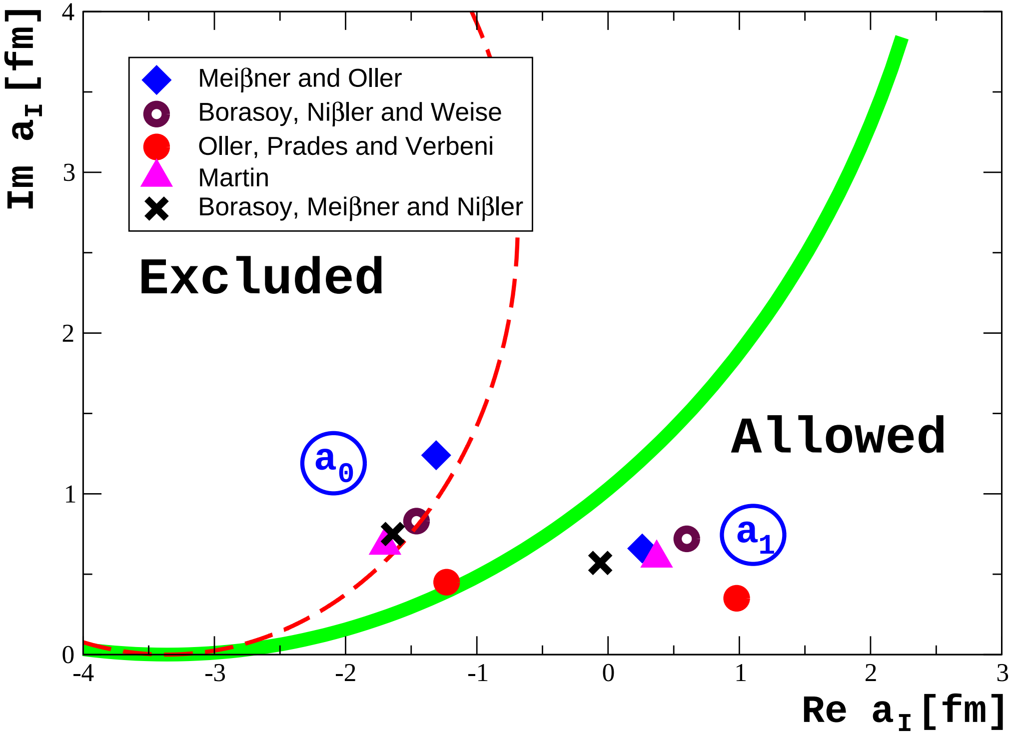
All scattering lengths are given in fm

ΔE (eV)	Exact	DGBT	MRR	MRR-resummed
$232 - i\,363$	$-0.50 + i\,1.01$	$-0.56 + i\,0.88$	$-0.45 + i\,1.01$	$-0.47 + i\,1.02$
$266 - i\,354$	$-0.60 + i\,1.01$	$-0.64 + i\,0.85$	$-0.54 + i\,1.00$	$-0.57 + i\,1.01$

Isospin breaking in the $\bar{K}N$ potential?

see also: Y. Yan, arXiv:0905.4818

Constraints imposed by kaonic hydrogen data



$$a_0 + a_1 + \frac{2q_0}{1 - q_0 a_p} a_0 a_1 - \frac{2a_p}{1 - q_0 a_p} = 0\,,$$

Im $a_I \geq 0$ (unitarity)

U.-G. Meißner, U. Raha and AR, EPJC 35 (2004) 349; EPJC 47 (2006) 473

Using kaonic deuteron data to determine a_0, a_1

$\hookrightarrow$ Extract $A_{\bar{K}d}$ from the experiment (SIDDHARTA at DAFNE)

$\hookrightarrow$ Relate a_0, a_1 *explicitly* to $A_{\bar{K}d}$ through multiple-scattering theory

Systematic uncertainty?

- Genuine uncertainty: 3-body force contributes up to a few %
- Isospin breaking: under control
- Going beyond static approximation:

  Potentially large corrections (up to 30%)

  Can be evaluated systematically in EFT

EFT approach to $\bar{K}d$ scattering

V. Baru, E. Epelbaum and AR, arXiv:0905.4249

- Different momentum scales $\longrightarrow$ multiple-scattering expansion

  $NN,\ \bar{K}NN$: *one-pion exchange*
  $\bar{K}N$: *two-pion exchange*

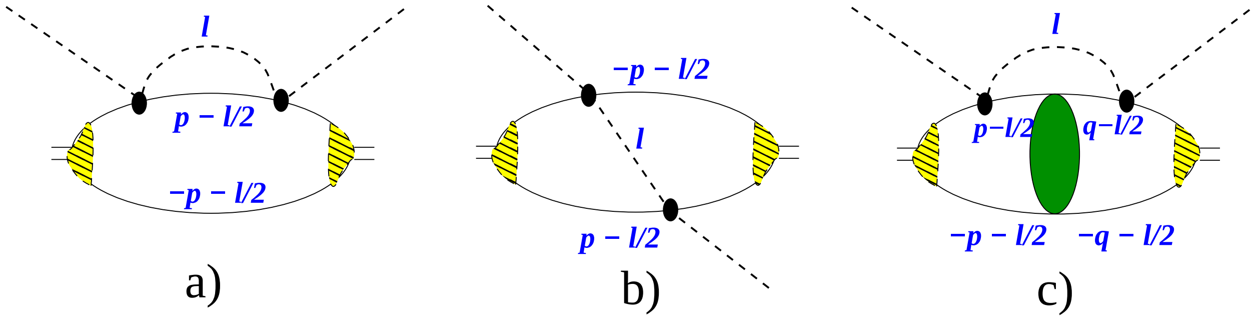
- The convergence of the series is controlled by $a \cdot \langle \frac{1}{r} \rangle \simeq 1$, where $\langle \frac{1}{r} \rangle \simeq 0.5$ fm^{-1}. S-wave scattering lengths are large due to the presence of the subthreshold $\Lambda\,(1405)$ resonance

$\longrightarrow$ Re-summation (is done in the *static* approximation)

- Exact solution of Faddeev equations:

$\longrightarrow$ Retardation effects moderate albeit $\xi = M_K/m_N \simeq 0.5$
$\longrightarrow$ Need to be understood in a systematic approach based on EFT

Second order



$$A_{\bar{K}d}^{\text{doubl. scatt.}} = \frac{8\pi\mu_d M_K}{\mu^2}\,(R_a + R_b + R_c)$$

$$R_i = R_i^{\text{stat}} + \xi^{1/2}R_i^{(1)} + \xi R_i^{(2)} + \xi^{3/2}R_i^{(3)} + \cdots$$

$\hookrightarrow$ Calculate $R_i^{(1)}, R_i^{(2)}, \ldots$, using *uniform expansion* method

Uniform expansion method

R. E. Mohr *et al*, Ann. Phys. 321 (2006) 225

see also M. Beneke and V. A. Smirnov, NPB 522 (1998) 321: *Threshold expansion*

Low-momentum regime ⟶ half-integer powers of ξ

$$\frac{\mathbf{l}^2}{2M_K} \sim \frac{\mathbf{p}^2}{2m_N} \quad \Rightarrow \quad \mathbf{l} \sim \sqrt{\xi}\mathbf{p}\,, \quad \mathbf{p} \sim \langle\frac{1}{r}\rangle$$

High-momentum regime ⟶ integer powers of ξ

$$\mathbf{l} \sim \mathbf{p} \sim \langle\frac{1}{r}\rangle$$

Intermediate regime

$$\sqrt{\xi}\mathbf{p} \ll \mathbf{l} \ll \mathbf{p}$$

↪ Expand the integrand in Taylor series in each region separately

Cancellation of leading corrections

$$R = R^{\text{stat}} + \cancel{\xi^{1/2} R^{(1)}} + \xi R^{(2)} + \xi^{3/2} R^{(3)} + \cdots$$

see also: G. Fäldt, Phys. Scripta 16 (1977) 81; V. Baru *et al*, PLB 589 (2004) 118

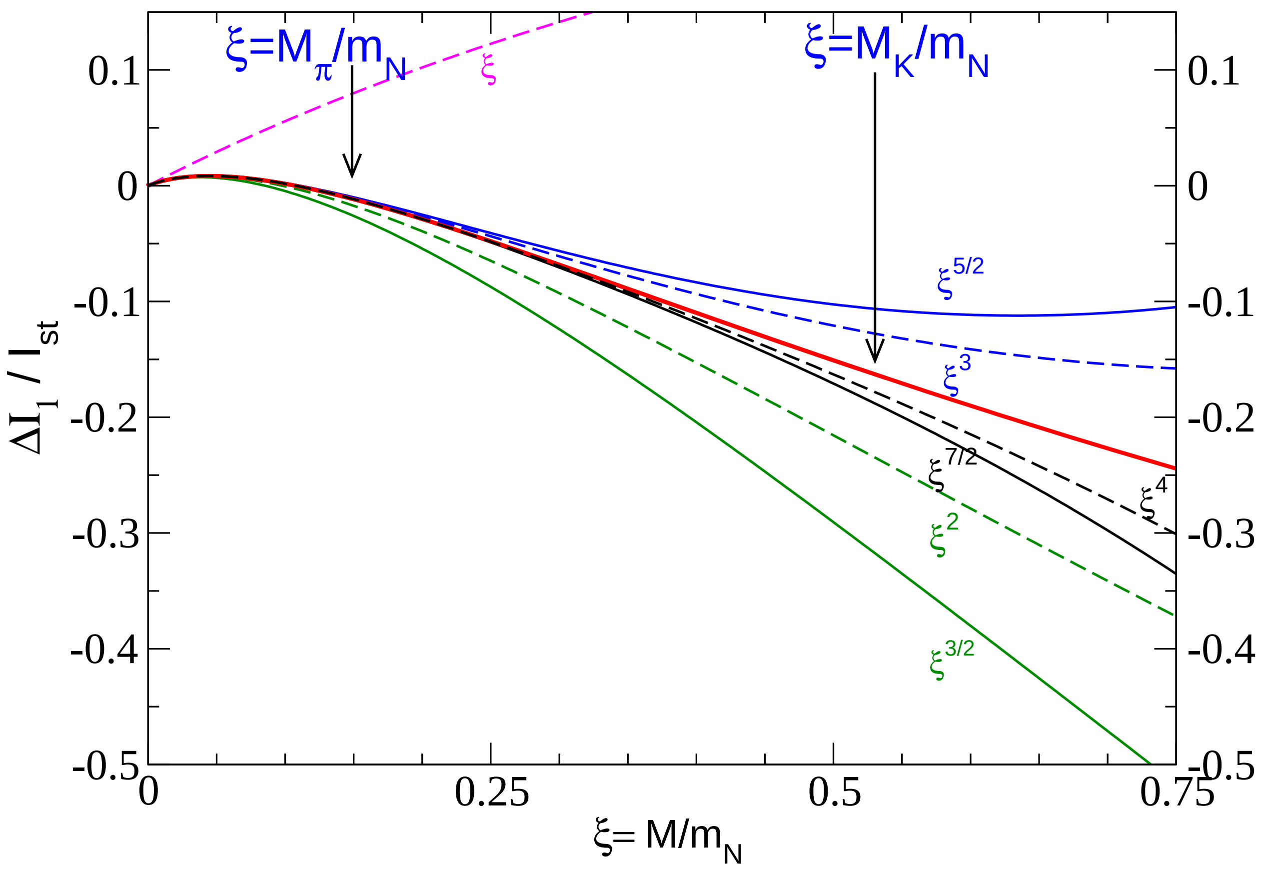
- Isospin-odd channel: Pauli-selection rules $\longrightarrow \quad R_-^{(1)} = 0$
- Isospin-even channel: at leading order in ξ,

$$R_+^{(1)} \sim \int \frac{d^3\mathbf{p} d^3\mathbf{q} d^3\mathbf{l}}{(2\pi)^6} \Psi(\mathbf{p}) \left(G_{NN}(\mathbf{p},\mathbf{q};E(\mathbf{l})) - \frac{\delta^3(\mathbf{p}-\mathbf{q})}{\mathbf{l}^2/2M_K} \right) \Psi(\mathbf{q})$$

$\hookrightarrow$ Vanishes at leading order due to the orthogonality of the bound-state and continuum wave functions

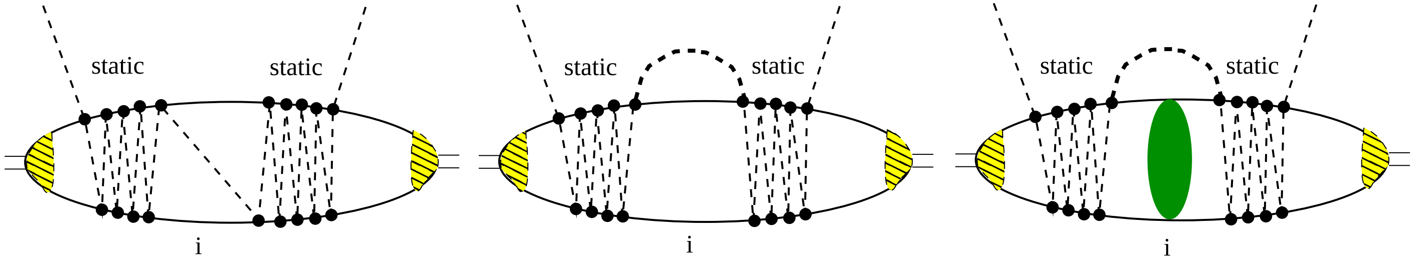
Convergence of the expansion

Corrections to the isospin-odd amplitude
(the kinematical factor $(1+\xi)^{-1}$ is not expanded)



Multiple-scattering series

V. Baru, E. Epelbaum and AR, in progress



- Assume retardation correction perturbative in contrast to static interactions
- The terms at $O(\xi^{1/2})$ do not vanish any more, but are numerically small (preliminary)

Conclusions

- $\pi\pi$ and πK atoms: theory ahead of the experiment
- πN and πd atoms: high-precision data available

$\longrightarrow$ Need to address isospin breaking in pionic deuterium at next-to-leading order

V. Baru, C. Hanhart, M. Hoferichter, B. Kubis, A. Nogga, D. Phillips, in progress

$\longrightarrow$ Isospin breaking in the hydrogen energy and width at $O(p^4)$

- $\bar{K}N$ and $\bar{K}d$ atoms: progress is foreseen both on experimental and theoretical sides:

$\longrightarrow$ Need to achieve a systematic description of the retardation corrections within the *EFT approach*

V. Baru, E. Epelbaum, AR, in progress