Neutron Spin Sum Rules and Spin Polarizabilities at low Q^2

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Sixth International Workshop on Chiral Dynamics

7 July 2009

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Introduction

- Experiment E97-110:
  - Precise measurement of moments of spin structure functions at low Q^2, 0.02 to 0.3 GeV^2 for the neutron and ^{3}He.

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  - Cover an unmeasured region of kinematics to test theoretical calculations (Chiral Perturbation Theory).
  - Data from experiment E94-010 covered the transition region (0.1 to 0.9 GeV^2) from non-perturbative to perturbative QCD.
  - Preliminary results are now available and final results are expected soon.

Inclusive Electron Scattering

Energy transfer:

$$\nu = E - E'$$

4-momentum transfer squared:

$$\vec{q} = \vec{k} - \vec{k}'$$

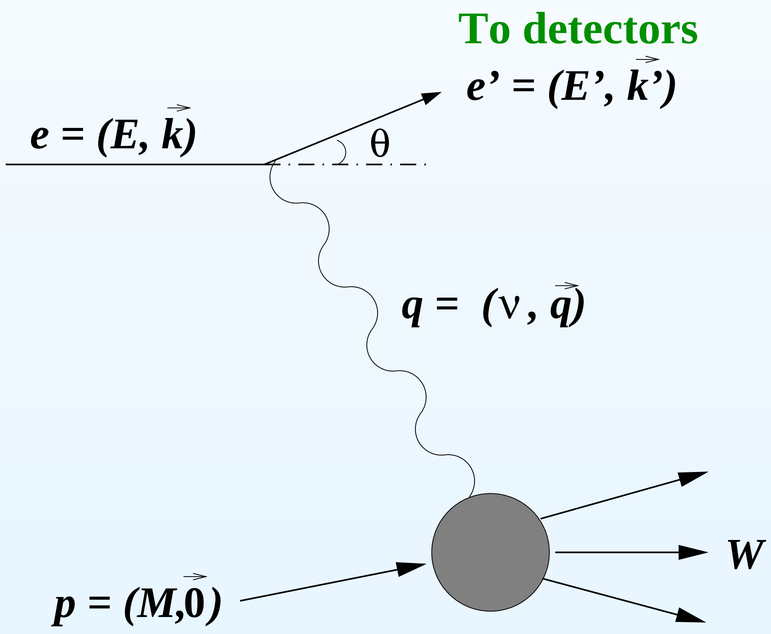
$$Q^2 = -q^2 = 4EE' \sin^2 \frac{\theta}{2}$$

Invariant Mass:

$$W^2 = M^2 + 2M\nu - Q^2$$

Bjorken variable:

$$x = \frac{Q^2}{2M\nu}$$



Inclusive Cross Sections

- structure functions:

  g_1 and g_2 (quark polarizations)

  or σ_{TT} and σ_{LT}

- Polarized cross sections:

$$\Delta\sigma_{\parallel} = \frac{d^2\sigma^{\downarrow\Uparrow}}{dE'd\Omega} - \frac{d^2\sigma^{\uparrow\Uparrow}}{dE'd\Omega} = K\left[(E + E'\cos\theta)\, g_1(x,Q^2) - \left(\frac{Q^2}{\nu}\right) g_2(x,Q^2)\right]$$

$$\Delta\sigma_{\perp} = \frac{d^2\sigma^{\downarrow\Rightarrow}}{dE'd\Omega} - \frac{d^2\sigma^{\uparrow\Rightarrow}}{dE'd\Omega} = KE'\sin\theta\left[g_1(x,Q^2) + \frac{2E}{\nu} g_2(x,Q^2)\right]$$

$$K = \frac{4\alpha^2}{M\nu Q^2}\frac{E'}{E}$$

$\downarrow\uparrow$ are for electron spin, $\Uparrow\Rightarrow$ are for target spin direction

Inclusive Cross Sections

- structure functions:

  g_1 and g_2 (quark polarizations)

  or σ_{TT} and σ_{LT}

- Virtual photon-nucleon polarized cross sections:

$$2\sigma_{TT}\left(x,Q^2\right) = \sigma_{1/2}\left(x,Q^2\right) - \sigma_{3/2}\left(x,Q^2\right)$$

$$= \frac{8\pi^2\alpha}{MK}\left[g_1\left(x,Q^2\right) - \frac{4M^2x^2}{Q^2}g_2\left(x,Q^2\right)\right]$$

$$\sigma_{LT}\left(x,Q^2\right) = \frac{4\pi^2\alpha}{MK}\left[g_1\left(x,Q^2\right) + g_2\left(x,Q^2\right)\right]$$

K: virtual photon flux

$\sigma_{1/2}$, $\sigma_{3/2}$: electroproduction cross sections

Gerasimov-Drell-Hearn (GDH) Sum Rule ($Q^2 = 0$)

$$I_{\rm GDH} = \int_{\nu_{\rm th}}^{\infty} \frac{\sigma_{\frac{1}{2}}(\nu) - \sigma_{\frac{3}{2}}(\nu)}{\nu} d\nu = -2\pi^2\alpha\left(\frac{\kappa}{M}\right)^2$$

- Circularly polarized photons incident on a longitudinally polarized spin-$\frac{1}{2}$ target.
- $\sigma_{\frac{1}{2}}$ ($\sigma_{\frac{3}{2}}$) photoabsorption cross section with photon helicity parallel (anti-parallel) to the target spin.
- The sum rule is related to the target's mass M and anomalous part of the magnetic moment κ.
- Solid theoretical predictions based on general principles.

GDH Measurements

The sum rule is valid for any target with definite spin-S.

	M [GeV]	Spin	κ	I_{GDH} [μ b]
Proton	0.938	$\frac{1}{2}$	1.79	–204.8
Neutron	0.940	$\frac{1}{2}$	-1.91	–233.2
Deuteron	1.876	1	-0.14	–0.65
Helium-3	2.809	$\frac{1}{2}$	-8.38	–498.0

- Proton sum rule was verified: Mainz, Bonn and LEGS.
- Measurements for the neutron (deuteron) are in progress.

Generalized GDH Integral ($Q^2 > 0$)

$$I(Q^2) = \int_{\nu_{\rm th}}^{\infty} \left[\sigma_{\frac{1}{2}}(\nu, Q^2) - \sigma_{\frac{3}{2}}(\nu, Q^2)\right] \frac{d\nu}{\nu}$$

$$\sigma_{1/2} - \sigma_{3/2} = \frac{8\pi^2\alpha}{MK}\left[g_1(\nu, Q^2) - \left(\frac{Q^2}{\nu^2}\right) g_2(\nu, Q^2)\right]$$

- Replace photoproduction cross sections with the corresponding electroproduction cross sections.
- The integral is related to the Compton scattering amplitudes: $S_1(Q^2)$ and $S_2(Q^2)$.

$$S_1(Q^2) = \frac{8}{Q^2}\int_0^1 g_1(x, Q^2)dx = \frac{8}{Q^2}\Gamma_1(Q^2)$$

X.-D. Ji and J. Osborne, J. Phys. **G27**, 127 (2001)

At $Q^2 = 0$, the GDH sum rule is recovered.

First moments of g_1 and g_2

$$\Gamma_1 = \int_0^1 g_1(x, Q^2)dx$$

$$\Gamma_2 = \int_0^1 g_2(x, Q^2)dx$$

- Γ_1 is closely related to generalized GDH integral as $Q^2 \to 0$.
- g_2 is suppressed at very low Q^2.

Bjorken Sum Rule ($Q^2 \to \infty$)

$$\Gamma_1^{\mathrm{p}} - \Gamma_1^{\mathrm{n}} = \frac{g_{\mathrm{A}}}{6}$$

J.D. Bjorken, Phys. Rev. **148**, 1467 (1966)

- g_{A} is the nucleon axial charge.
- The sum rule has been confirmed to 10%.

Importance of the Generalized GDH Sum Rule

Lattice QCD

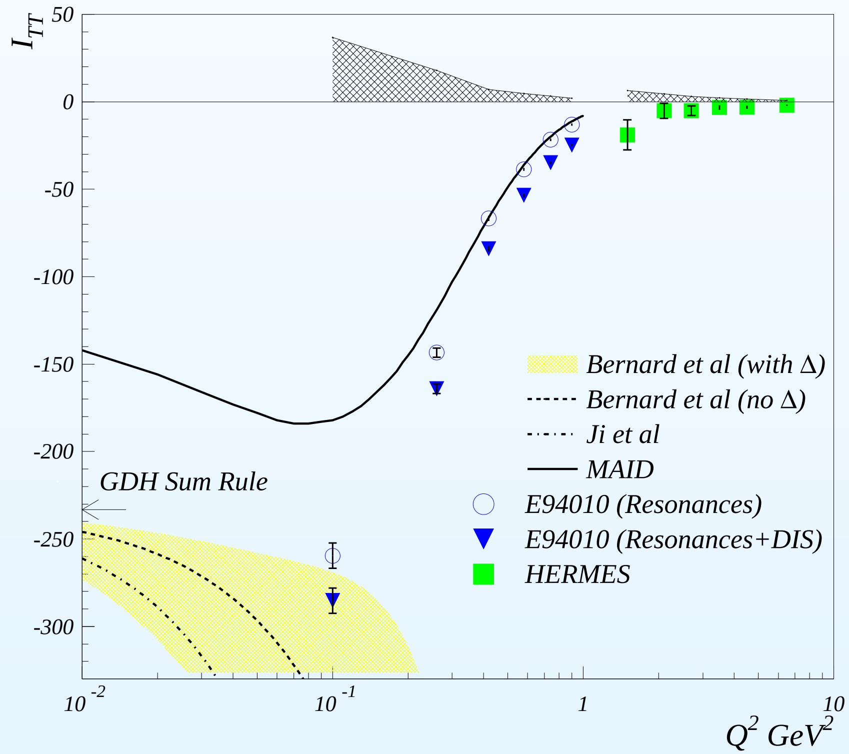
χPT OPE pQCD

$Q^2 = 0$ Hadronic → $Q^2 = \infty$ Partonic

- Constrained at the two ends of the Q^2 spectrum by known sum rules: GDH ($Q^2 = 0$) and Bjorken ($Q^2 \to \infty$).
- Generalized GDH Integral is calculable at any Q^2.
- Compare theoretical calculations to experimental measurements over the measurable Q^2 range.
- Tool to study non-perturbative QCD, while starting on known theoretical grounds (pQCD).

Hall A Neutron GDH Published Results

Neutron



M. Amarian *et al.*, PRL **89**, 242301 (2002)

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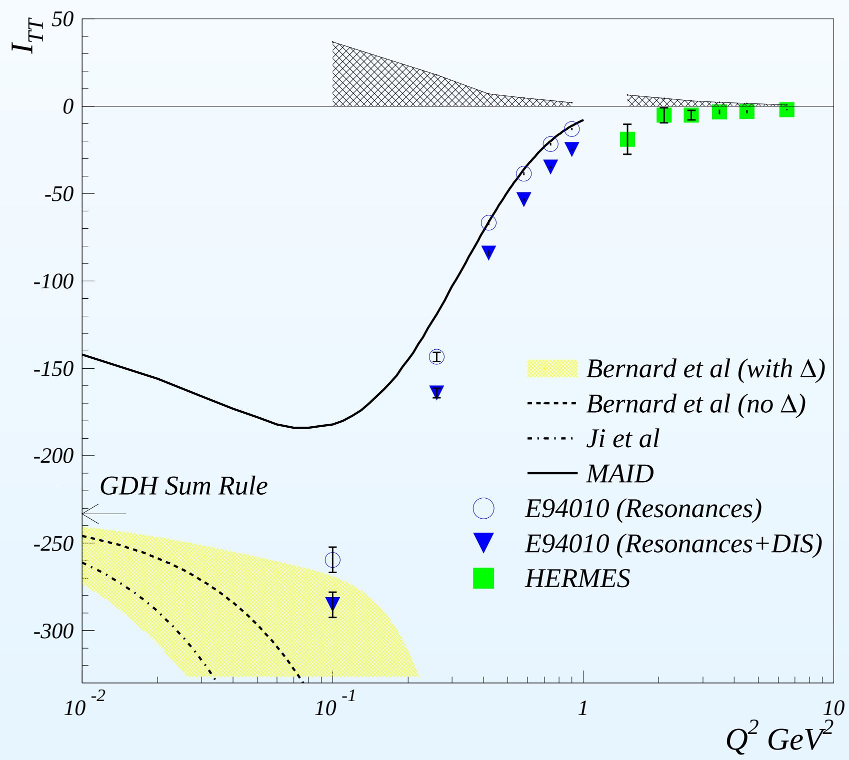
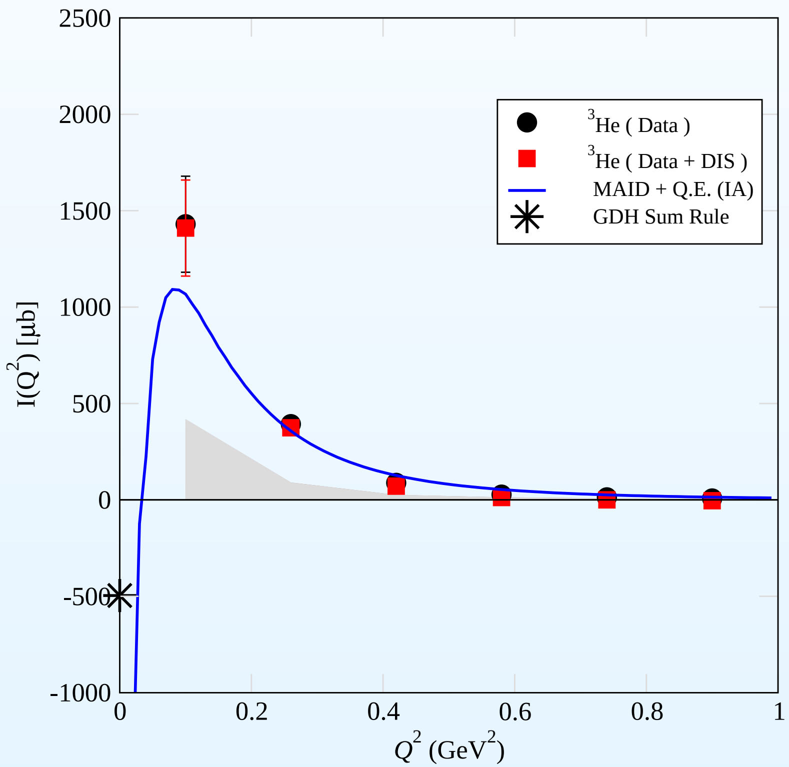
Neutron



M. Amarian *et al.*, PRL **89**, 242301 (2002)

Helium-3

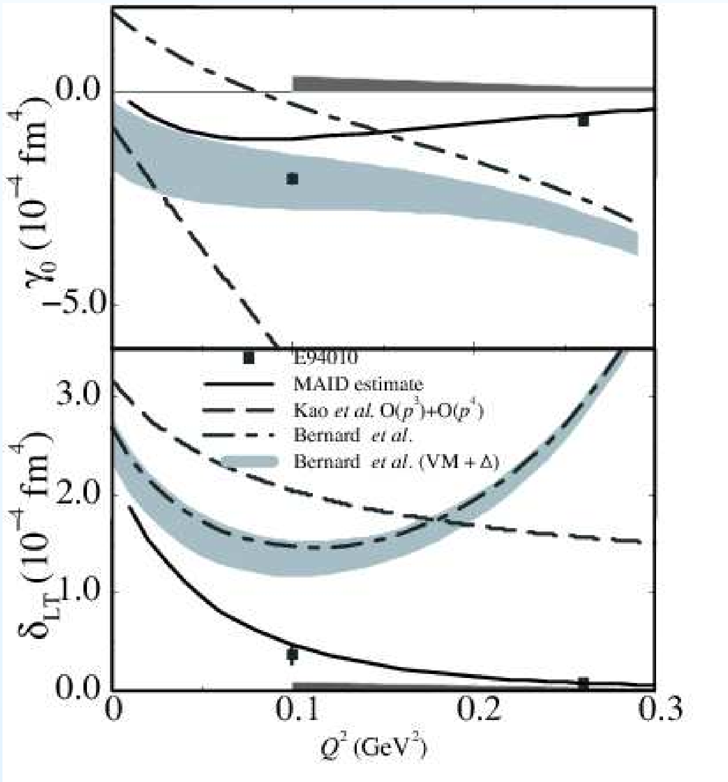


K. Slifer *et al.*, PRL **101**, 022303 (2008).

Neutron Spin Polarizabilities

$$\gamma_0 = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left(g_1 - \frac{4M^2}{Q^2} x^2 g_2\right) dx$$

$$\delta_{\rm LT} = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left(g_1 + g_2\right) dx$$

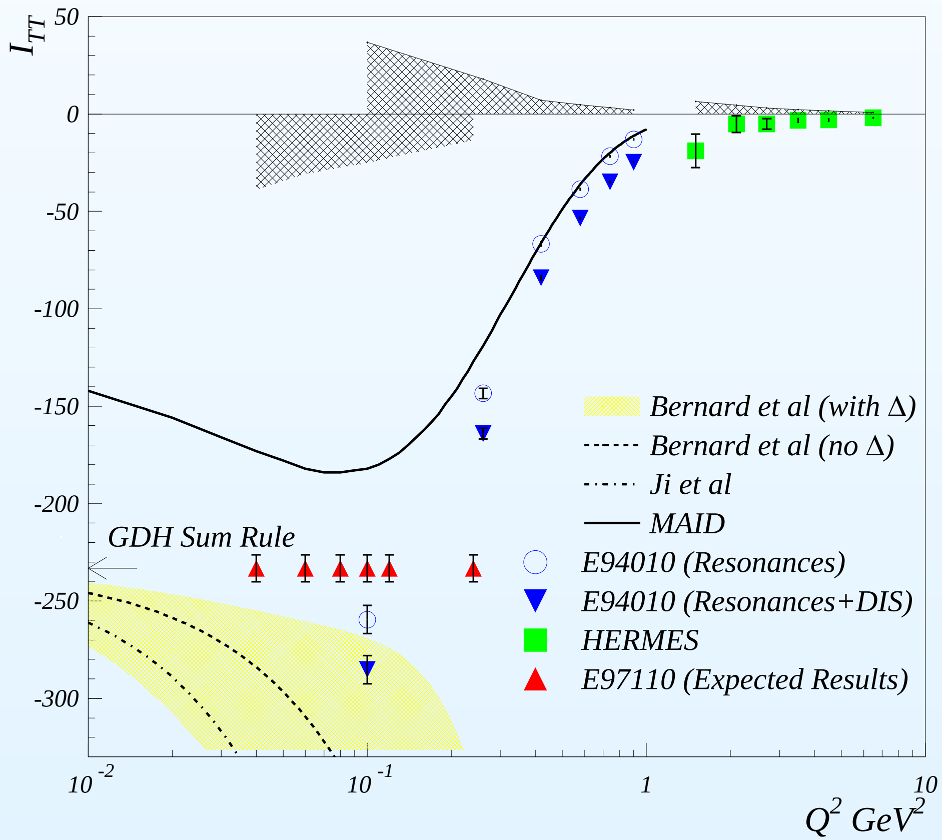


M. Amarian *et al.* , PRL **93**, 152301 (2004)

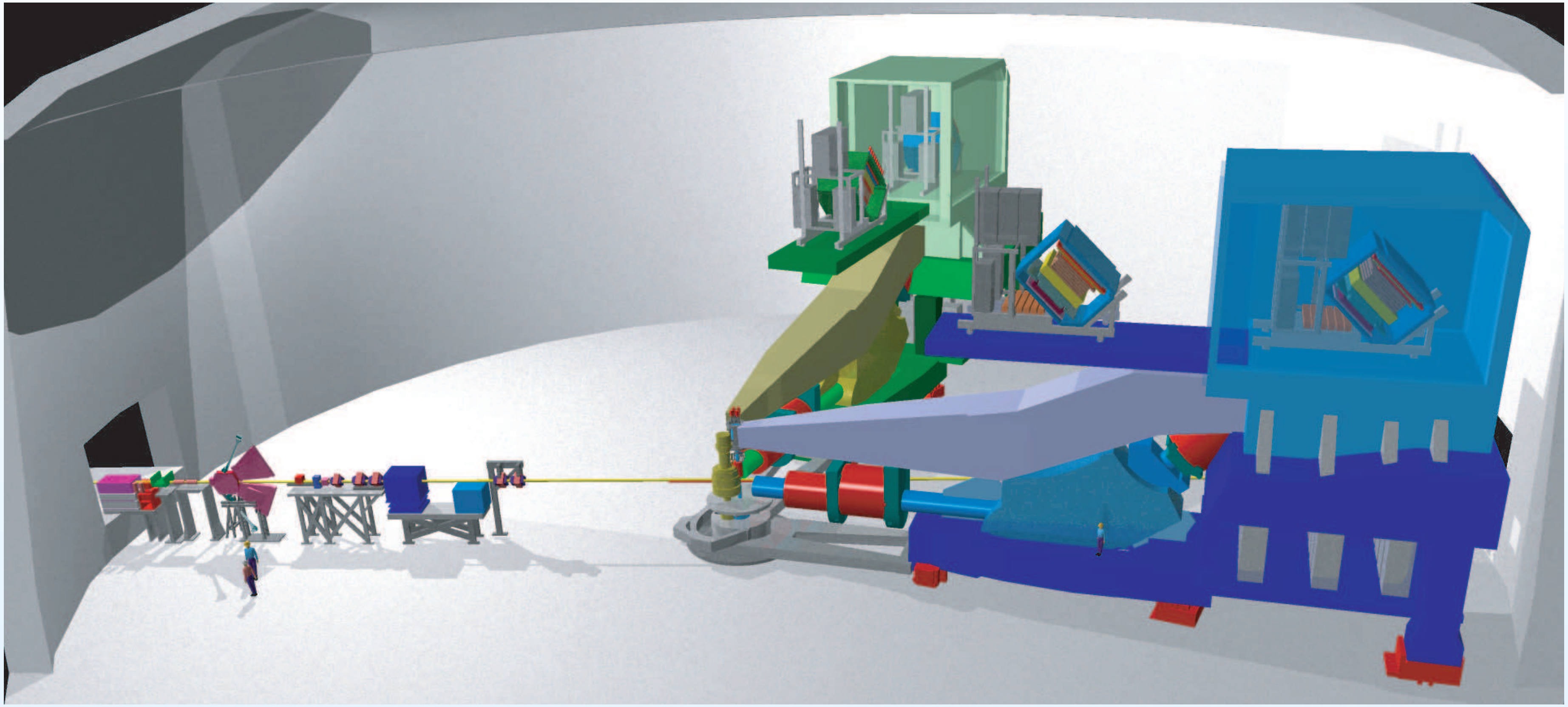
Experiment E97-110

Precise measurement of generalized GDH integral at low Q^2, 0.02 to 0.3 GeV^2

- Ran in spring and summer 2003
- Inclusive experiment: $^3\overrightarrow{\mathrm{He}}(\vec{e},e')X$
  - ⇒ Scattering angles of 6° and 9°
  - ⇒ Polarized electron beam:
    $\langle P_{\mathrm{beam}} \rangle$ = 75%
  - ⇒ Pol. ^{3}He target (para & perp):
    $\langle P_{\mathrm{targ}} \rangle$ = 40%
- Measured polarized cross-section differences

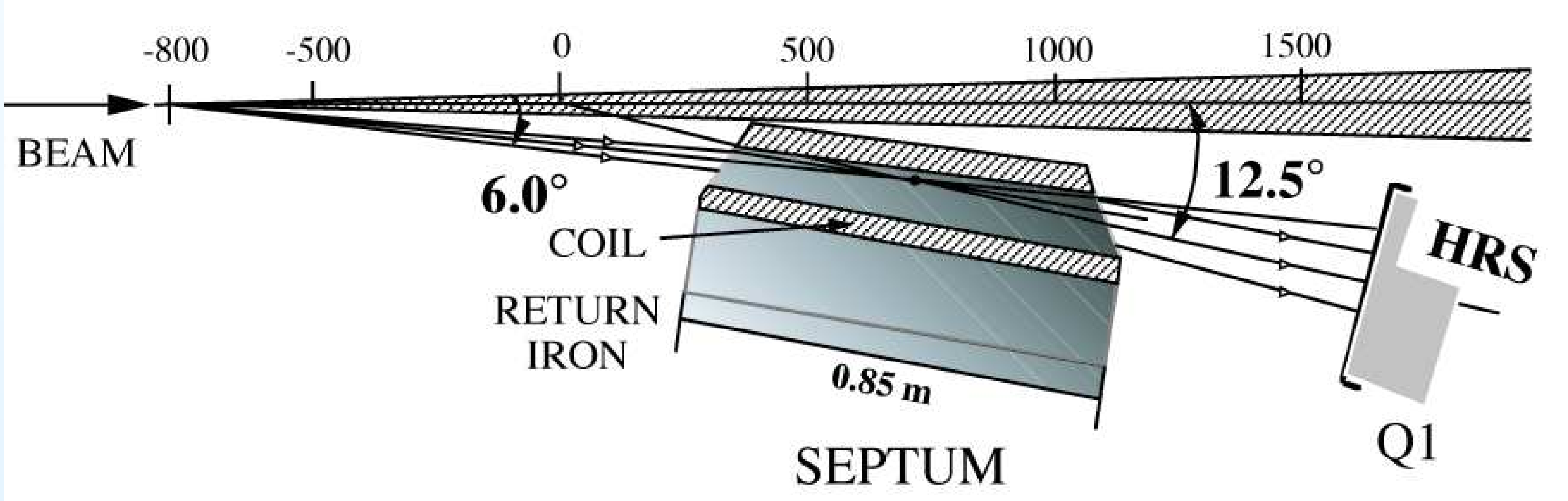


Experimental Setup

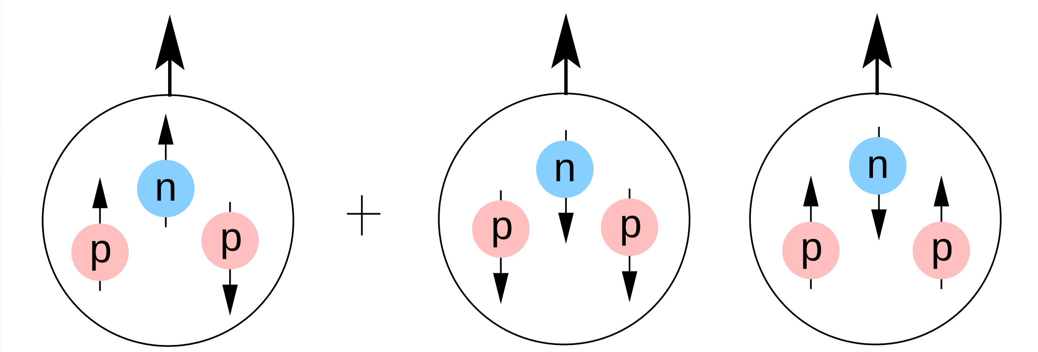


New Bending Magnet

- Low Q^2 requires forward angles.
- Minimum spectrometer angle is 12.5°.



^{3}He as an Effective Polarized Neutron Target



S state ($\sim$88.2%) | D state ($\sim$9.8%) | S' state ($\sim$1.4%)

P_n = 86% and P_p = -2.8%

J.L. Friar *et al.*, PRC **42**, (1990) 2310

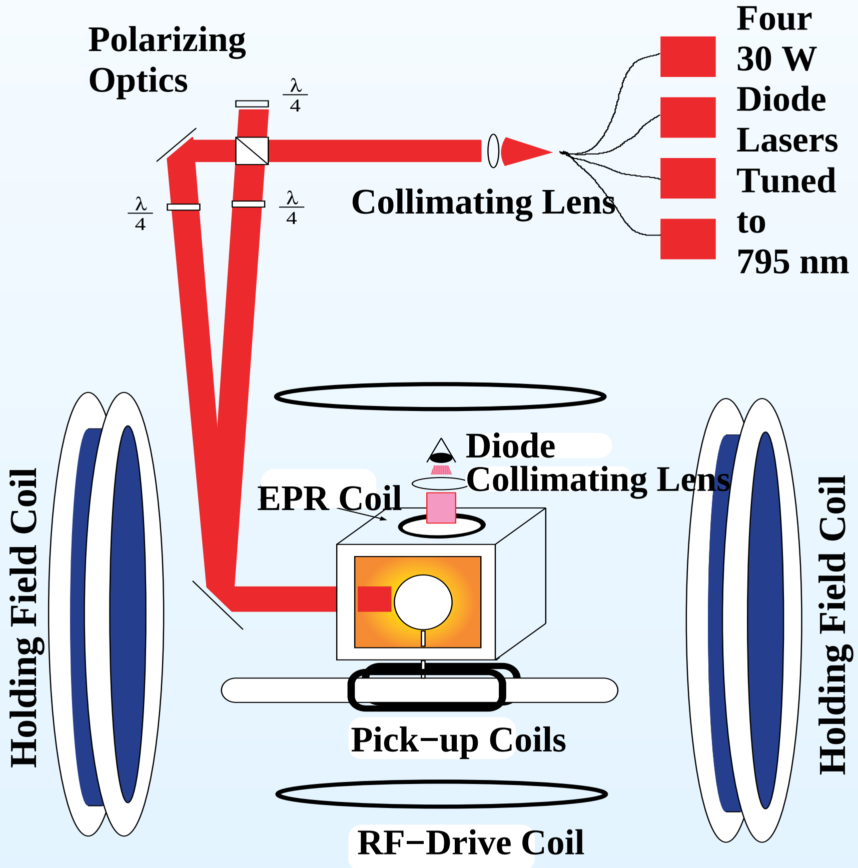
Extraction of Neutron Results

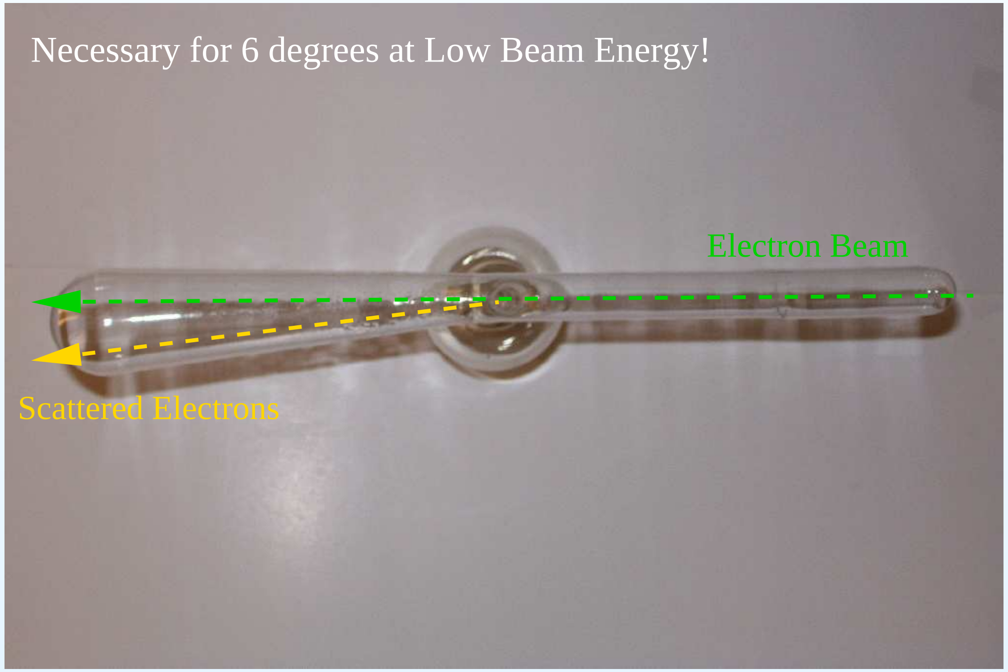
$$\Gamma_1^\mathrm{n}(Q^2) = \frac{1}{P_\mathrm{n}}\left[\Gamma_1^{^3\mathrm{He}}(Q^2) - 2P_\mathrm{p}\Gamma_1^\mathrm{p}(Q^2)\right]$$

C. Ciofi degli Atti & S. Scopetta, PLB **404**, (1997) 223

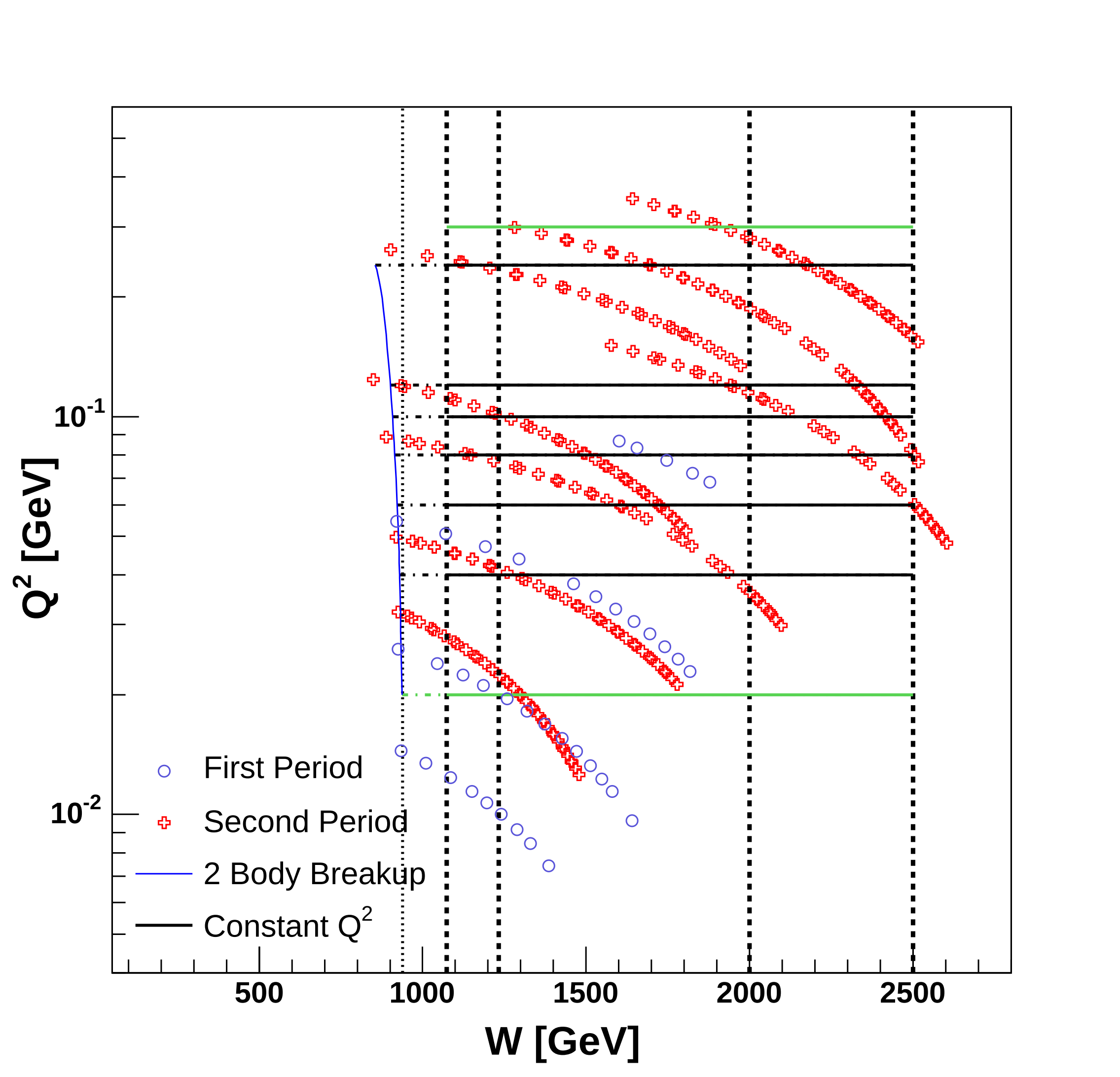
Polarized ^{3}He System

- Both longitudinal and transverse configurations.
- Two independent polarimetries: NMR and EPR.





Kinematic Coverage and Interpolation



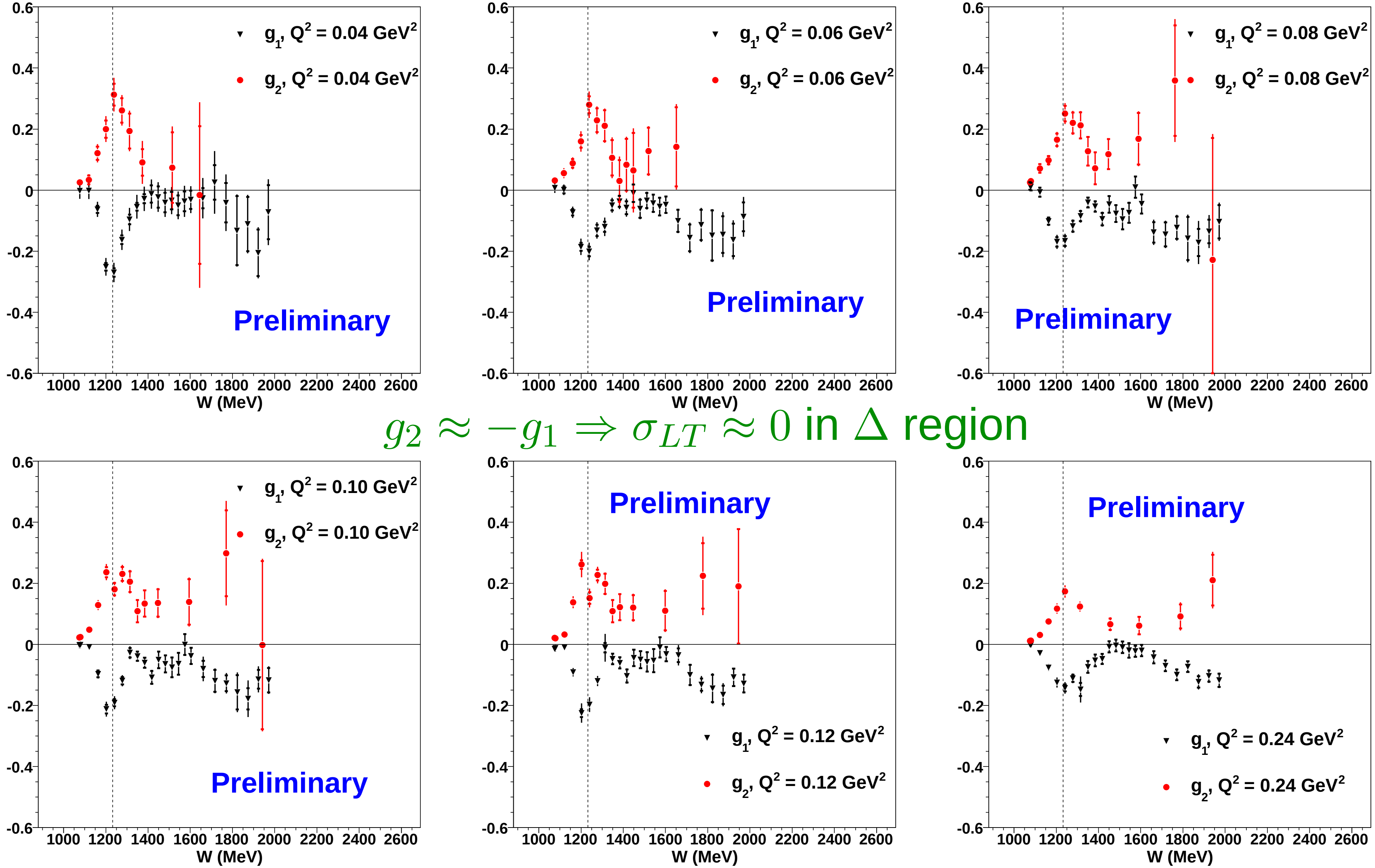
Six constant Q^2 points: 0.04, 0.06, 0.08, 0.1, 0.12 and 0.24 GeV^2.

Constant Q^2 Interpolation and Integral Extraction

Procedure:

- First interpolate to constant W for each energy.
- Second interpolation with respect to Q^2.
- Integrals formed from W = 1073 GeV to 2000 GeV.
- We could use our own data above W = 2000 GeV.
- DIS contribution included up to W = $\sqrt{1000}$ using Thomas and Bianchi parameterization.
- Neutron extraction performed using calculation from Scopetta and Ciofi degli Atti for $Q^2 >= 0.1$ GeV^2.
- $Q^2 < 0.1$ GeV^2 use effective polarization technique (difference $\sim$ 5–10%).

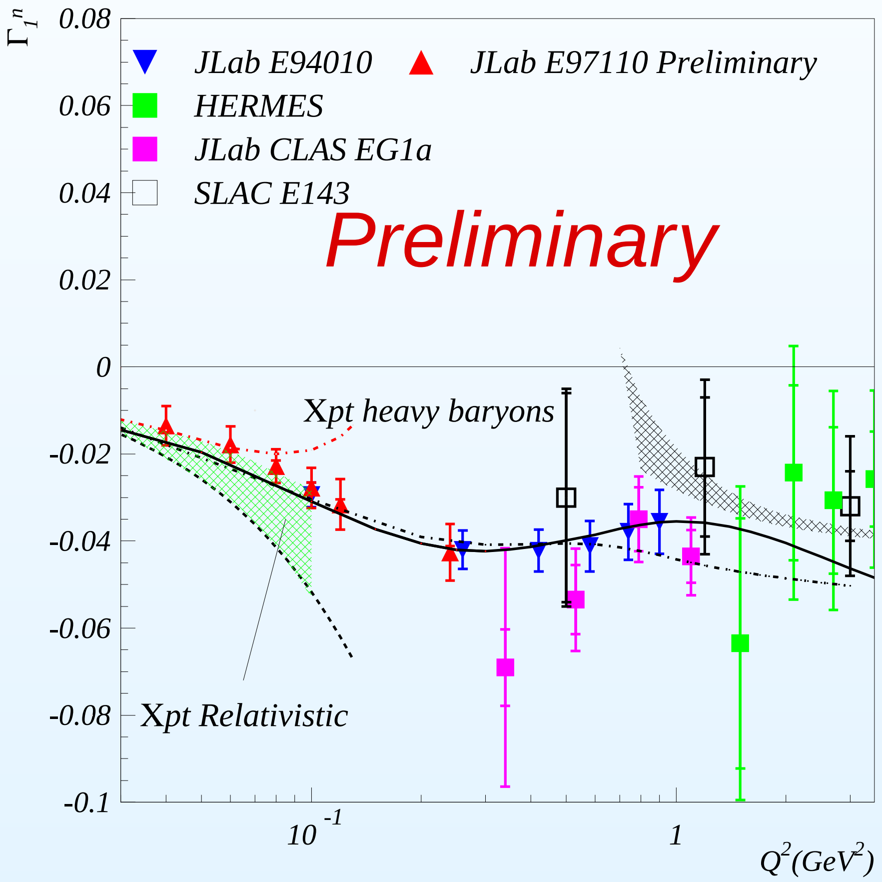
^{3}He - g_1, g_2 versus W at constant Q^2



$g_2 \approx -g_1 \Rightarrow \sigma_{LT} \approx 0$ in Δ region

First Moment of g_1

$$\Gamma_1 = \int_0^{x_0} g_1(x, Q^2) dx$$



Xpt - Chiral Perturbation Theory

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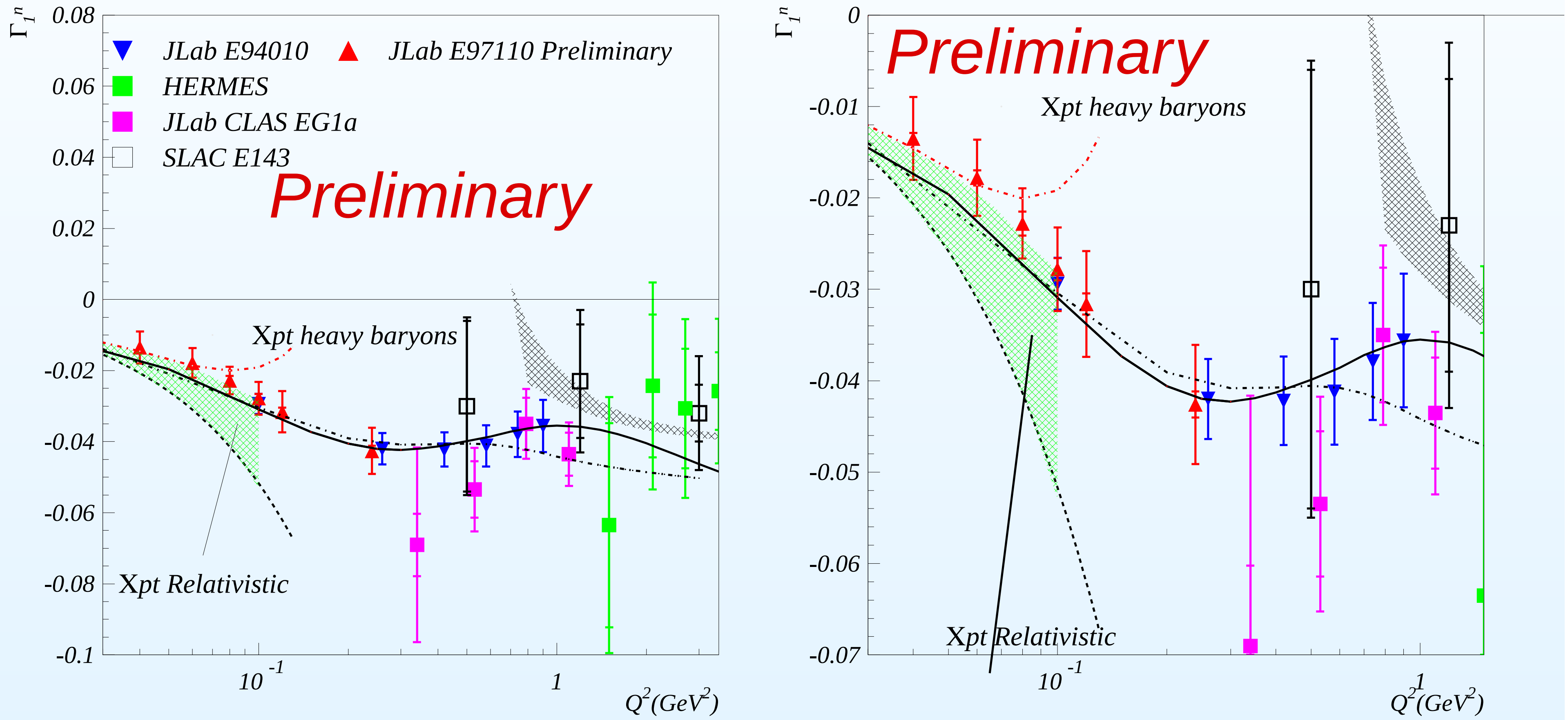
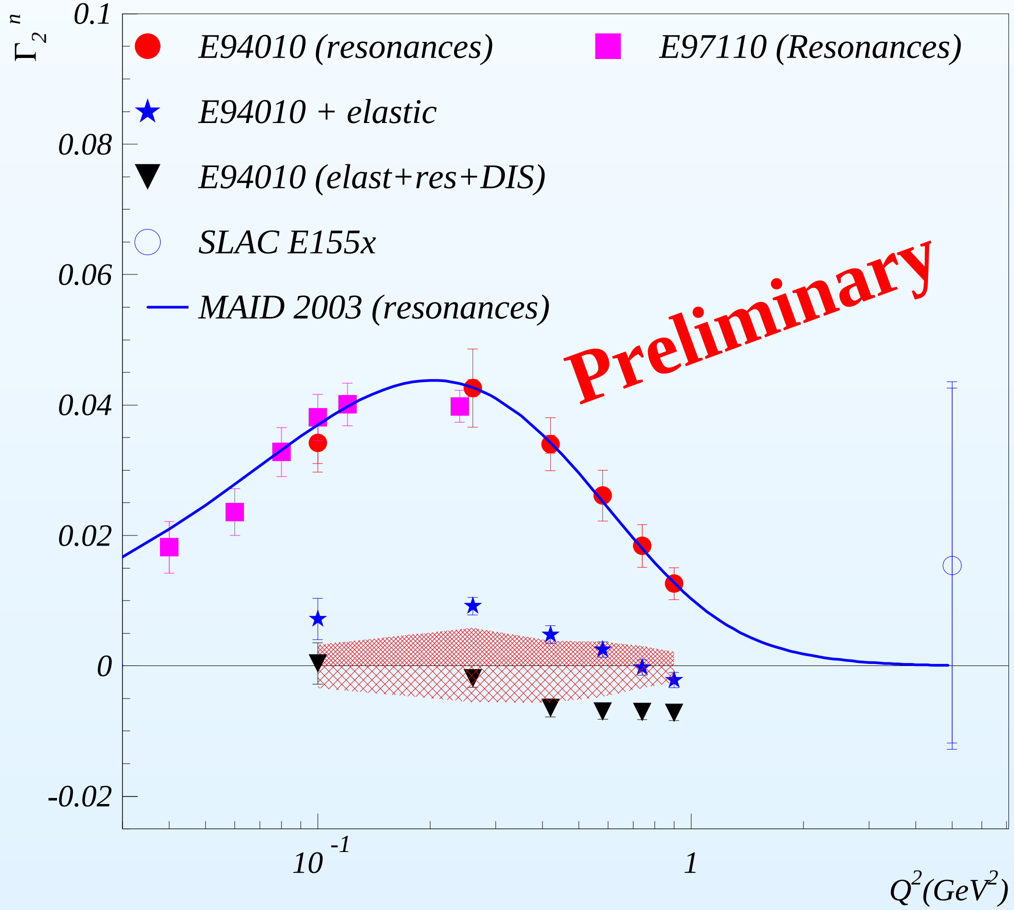


Xpt - Chiral Perturbation Theory

First Moment of g_2

$$\Gamma_2^n(Q^2) = \int_0^1 g_2(x, Q^2)dx = 0$$

Burkhardt-Cottingham Sum Rule



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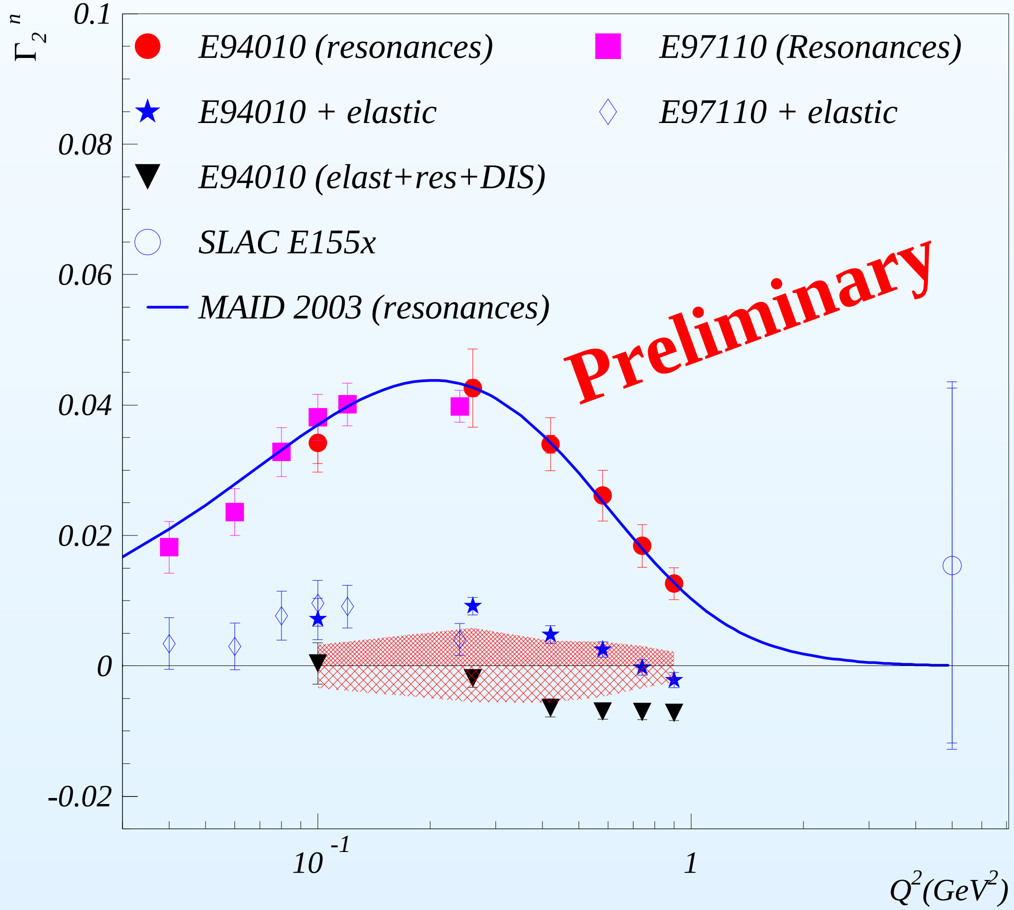

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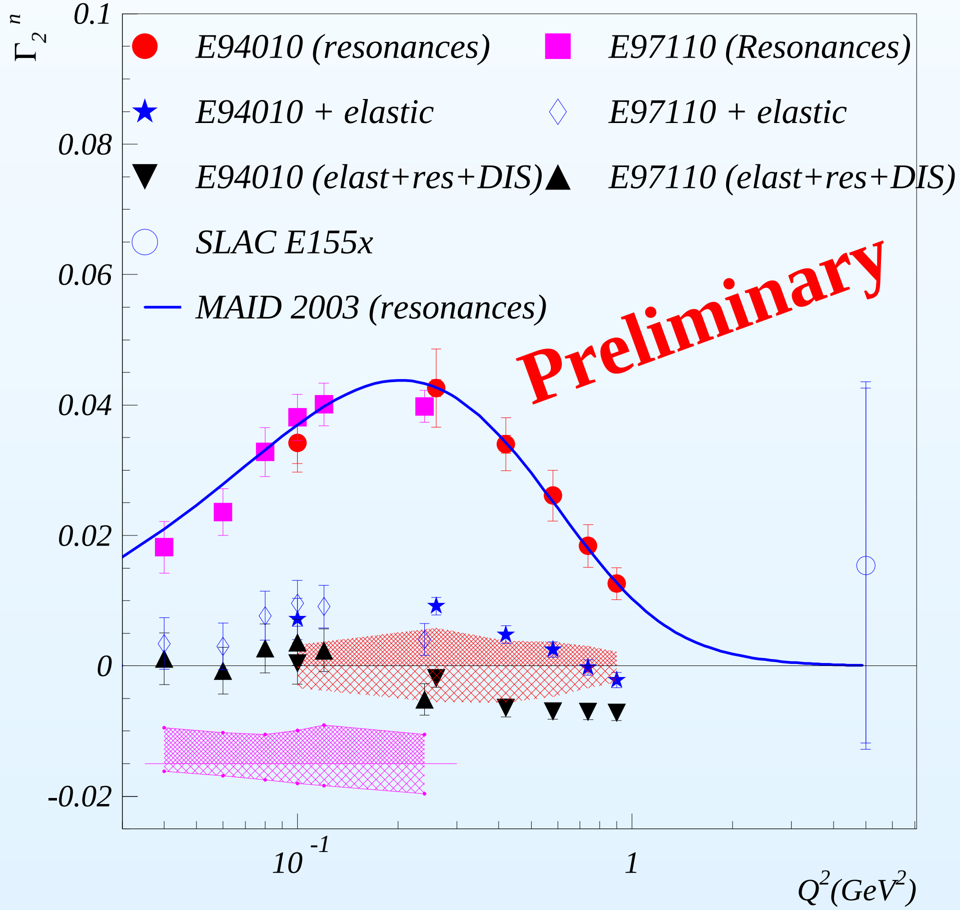
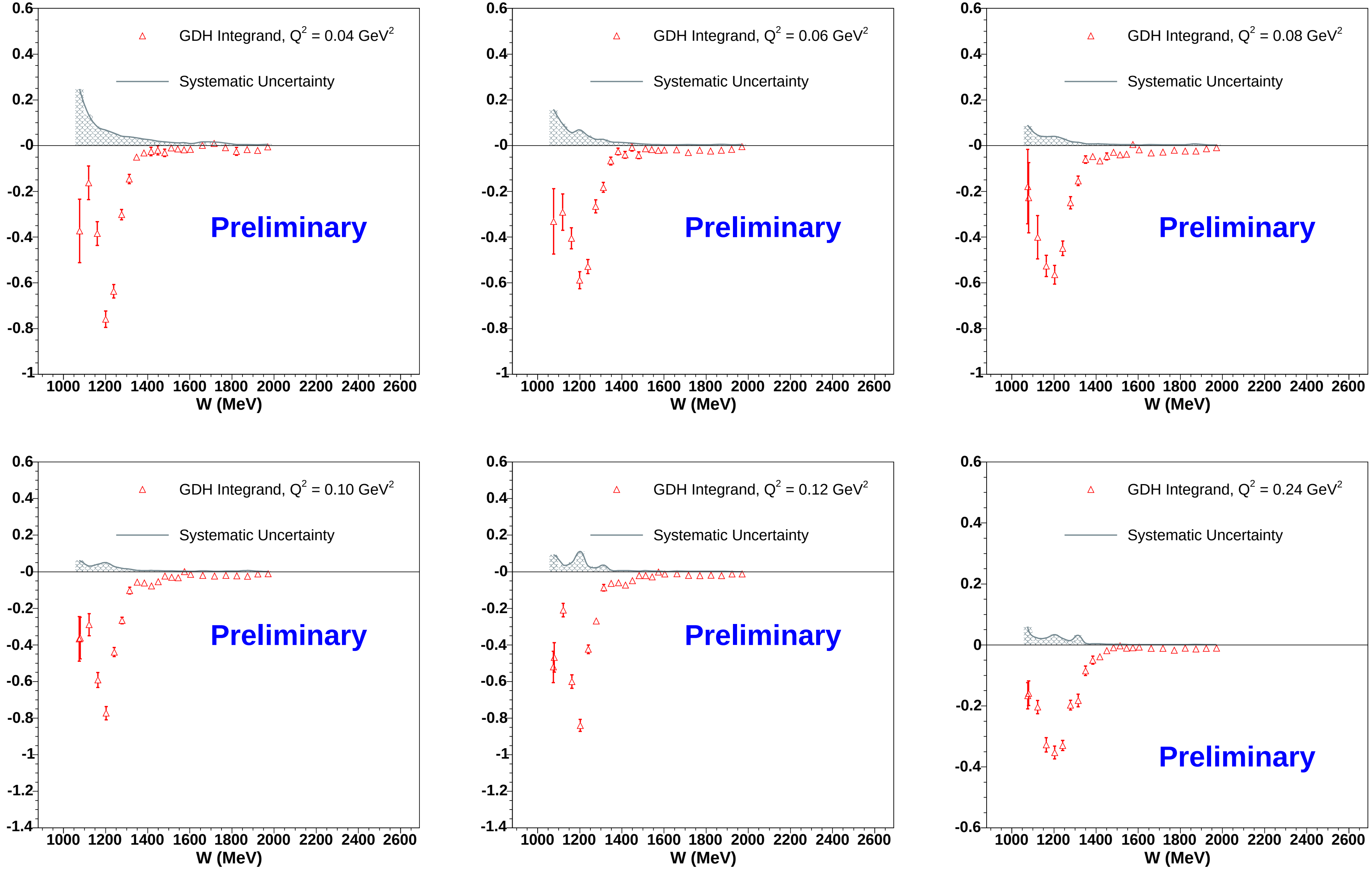


^{3}He - $\frac{\sigma_{TT}}{\nu}$ versus W at constant Q^2



Summary and Conclusion

- The GDH integral is an important tool that can be used to study nucleon spin structure over the full Q^2 range:
  - in particular, the transition from **perturbative QCD** to **nonperturbative QCD.**
- Experiment E97-110 provides precision data for moments of the spin structure functions at low Q^2: 0.02 to 0.3 [GeV/c]^2
- Preliminary results of the the neutron moments are available and work is in progress to finalize the systematic effects.
- These data provide a precise-benchmark test of Chiral Perturbation Theory calculations at a Q^2 where they are expected to be valid.
- Expect final neutron results soon.

The E97-110 Collaboration

S. Abrahamyan, K. Aniol, D. Armstrong, T. Averett, S. Bailey, P. Bertin, W. Boeglin, F. Butaru, A. Camsonne, G.D. Cates, G. Chang, **J.P. Chen**, Seonho Choi, E. Chudakov, L. Coman, J. Cornejo, B. Craver, F. Cusanno, R. De Leo, C.W. de Jager, **A. Deur**, K.E. Ellen, R. Feuerbach, M. Finn, S. Frullani, K. Fuoti, H. Gao, **F. Garibaldi**, O. Gayou, R. Gilman, A. Glamazdin, C. Glashausser, J. Gomez, O. Hansen, D. Hayes, B. Hersman, D. W. Higinbotham, T. Holmstrom, T.B. Humensky, C. Hyde-Wright, H. Ibrahim, M. Iodice, X. Jiang, L. Kaufman, A. Kelleher, W. Kim, A. Kolarkar, N. Kolb, W. Korsch, K. Kramer, G. Kumbartzki, L. Lagamba, G. Laveissiere, J. LeRose, D. Lhuillier, R. Lindgren, N. Liyanage, B. Ma, D. Margaziotis, P. Markowitz, K. McCormick, Z.E. Meziani, R. Michaels, B. Moffit, P. Monaghan, S. Nanda, J. Niedziela, M. Niskin, K. Paschke, M. Potokar, A. Puckett, V. Punjabi, Y. Qiang, R. Ransome, B. Reitz, R. Roche, A. Saha, A. Shabetai, **J. Singh**, S. Sirca, K. Slifer, R. Snyder, P. Solvignon, R. Stringer, R. Subedi, **V. Sulkosky**, W.A. Tobias, P. Ulmer, G. Urciuoli, A. Vacheret, E. Voutier, K. Wang, L. Wan, B. Wojtsekhowski, S. Woo, H. Yao, **J. Yuan**, X. Zheng, L. Zhu

and the Jefferson Lab Hall A Collaboration

Extra Slides

Systematic Uncertainties

Source	Systematic Uncertainty		
Angle	6°	9°	3.775 GeV, 9°
Target density	2.0%		
Acceptance	5.0%	5.0%	15.0%
VDC efficiency	3.0%	2.5%	2.5%
Charge	1.0%		
PID efficiency	$<$ 1.0%		
$\delta\sigma_{\mathrm{raw}}$	6.4%	6.2%	15.5%
Nitrogen dilution	0.2–0.5%		
$\delta\sigma_{\mathrm{exp}}$	6.5%	6.3%	15.5%
Beam Polarization	3.5%		
Target Polarization	7.5%		
Radiative Corrections*	20% (40% for $Q^2 \leq$ 0.08)		
Total on $\Delta\sigma$	12.1%	12.0%	18.6%

$*$ Radiative correction uncertainty $\approx$ 6% in delta region

GDH Derivation for Real Photons

- Begin with the spin dependent part of the forward Compton amplitude, S_1
- Use the following dispersion relation and three assumptions:

$$\mathrm{Re}\ S_1(\nu) = \frac{2\nu}{\pi} \int_{\nu_{\rm th}}^{\infty} d\nu' \frac{\mathrm{Im}\ S_1(\nu')}{\nu'^2 - \nu^2}$$

- Optical Theorem: $\mathrm{Im}\ S_1(\nu) = \frac{\nu}{8\pi}\sigma_{TT}(\nu)$
- Low Energy Theorem: $\mathrm{Re}\ S_1(\nu) = -\frac{e^2\kappa^2}{8\pi M^2}\nu$
- Unsubstracted Dispersion Relation: assumption is convergence of the dispersion integral.

$$I_{\rm GDH} = \int_{\nu_{\rm th}}^{\infty} \frac{\sigma_{\frac{1}{2}}(\nu) - \sigma_{\frac{3}{2}}(\nu)}{\nu} d\nu = -2\pi^2\alpha\left(\frac{\kappa}{M}\right)^2$$

Chiral Symmetry

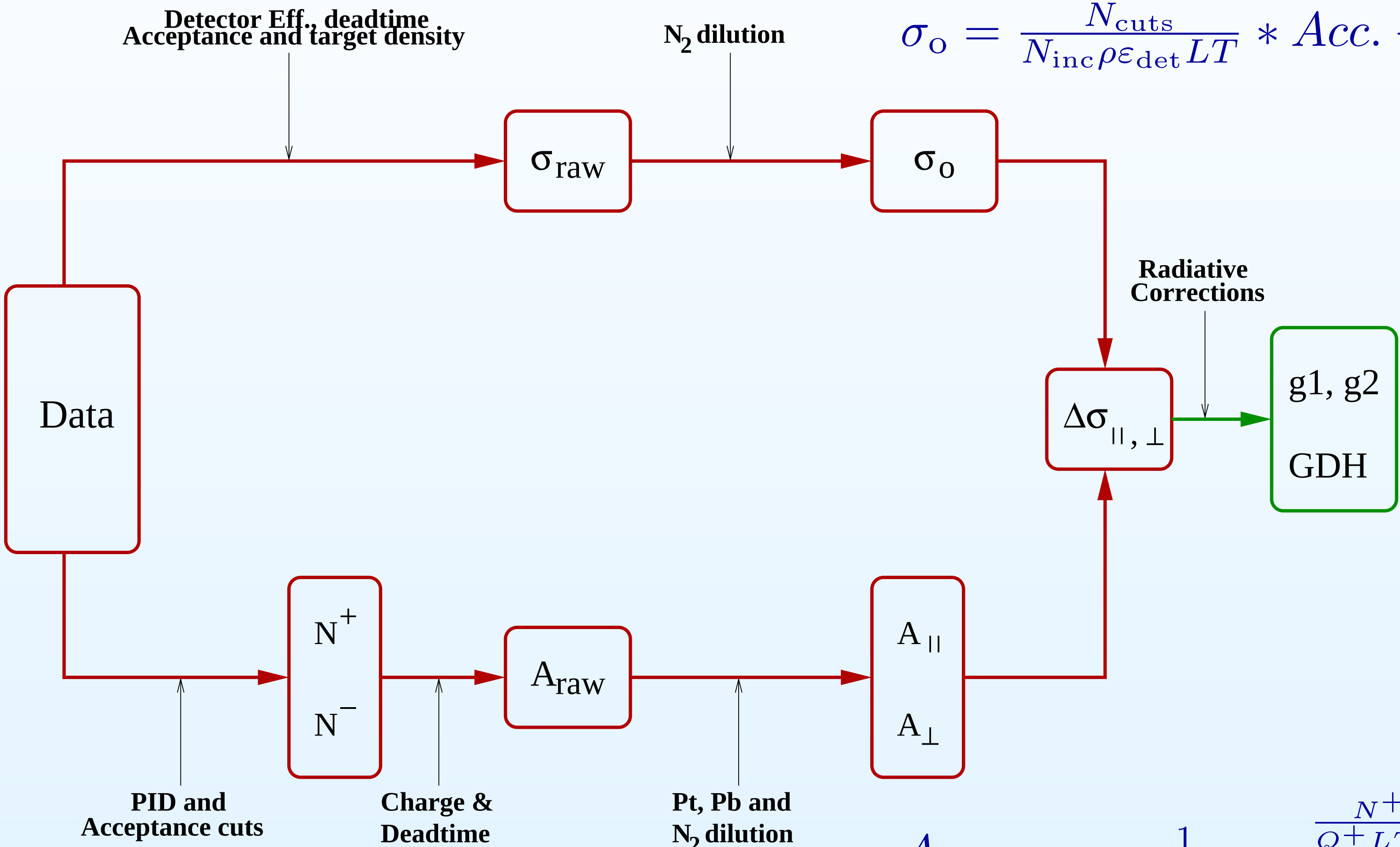
$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4g^2} G^{\alpha}_{\mu\nu} G^{\mu\nu}_{\alpha} + \bar{q} i \gamma^{\mu} D_{\mu} q - \bar{q} \mathcal{M} q$$
$$\mathcal{L}_{\mathrm{QCD}} = \mathcal{L}_0 + \mathcal{L}_{sb}$$

- Consider the limit where the light quark masses vanish.
- For massless fermions, chirality (handedness) is identical to a particle's helicity.
- Extra symmetry to the Lagrangian and obtain left and right handed quark fields.

$$q_{L,R} = \frac{1}{2}(1 \mp \gamma_5) q \,,$$

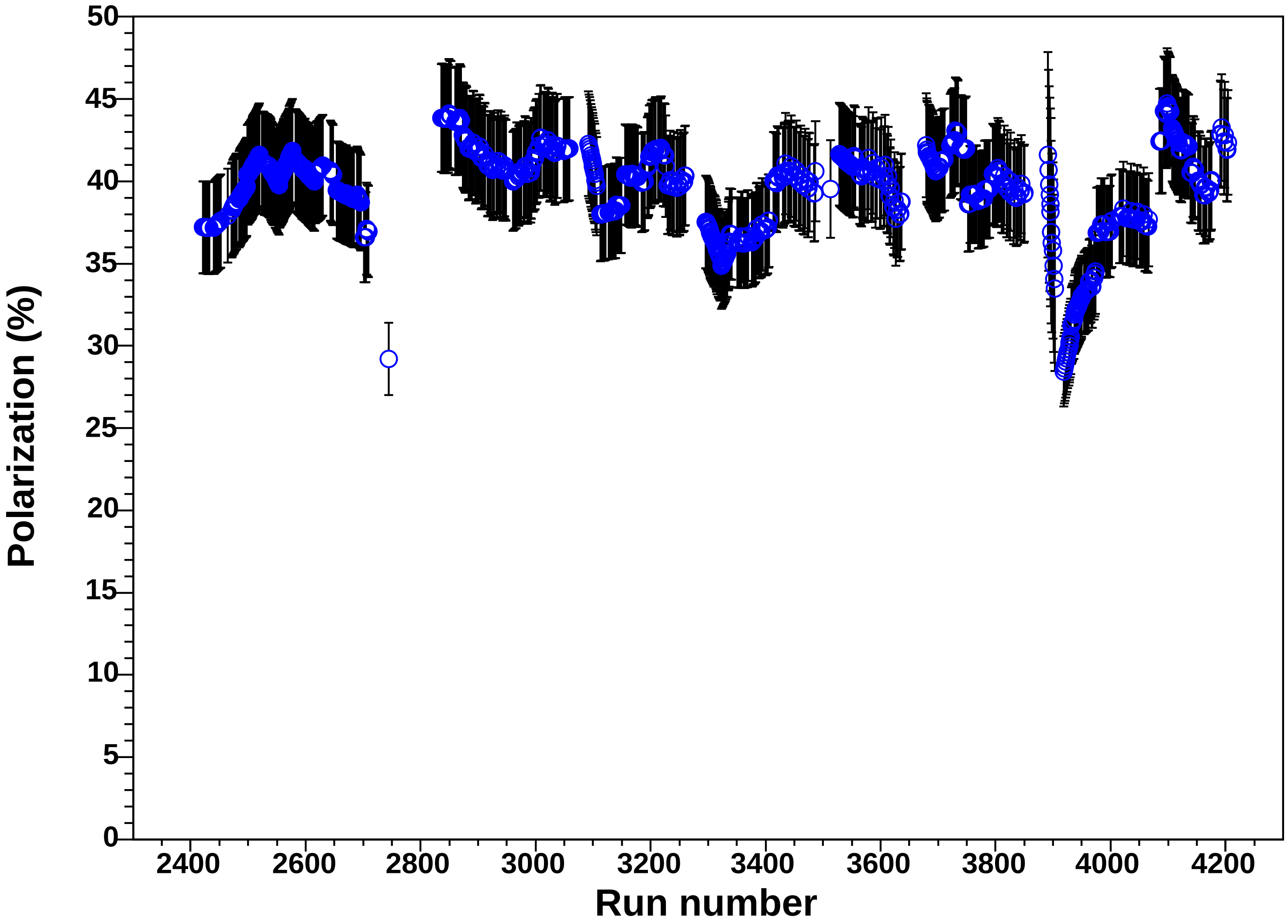
Analysis Procedure

$$\sigma_{\rm o} = \frac{N_{\rm cuts}}{N_{\rm inc}\rho\varepsilon_{\rm det}LT} * Acc. - \frac{2\rho_{\rm N_2}}{\rho + \rho_{\rm N_2}}\sigma_{\rm N_2}$$

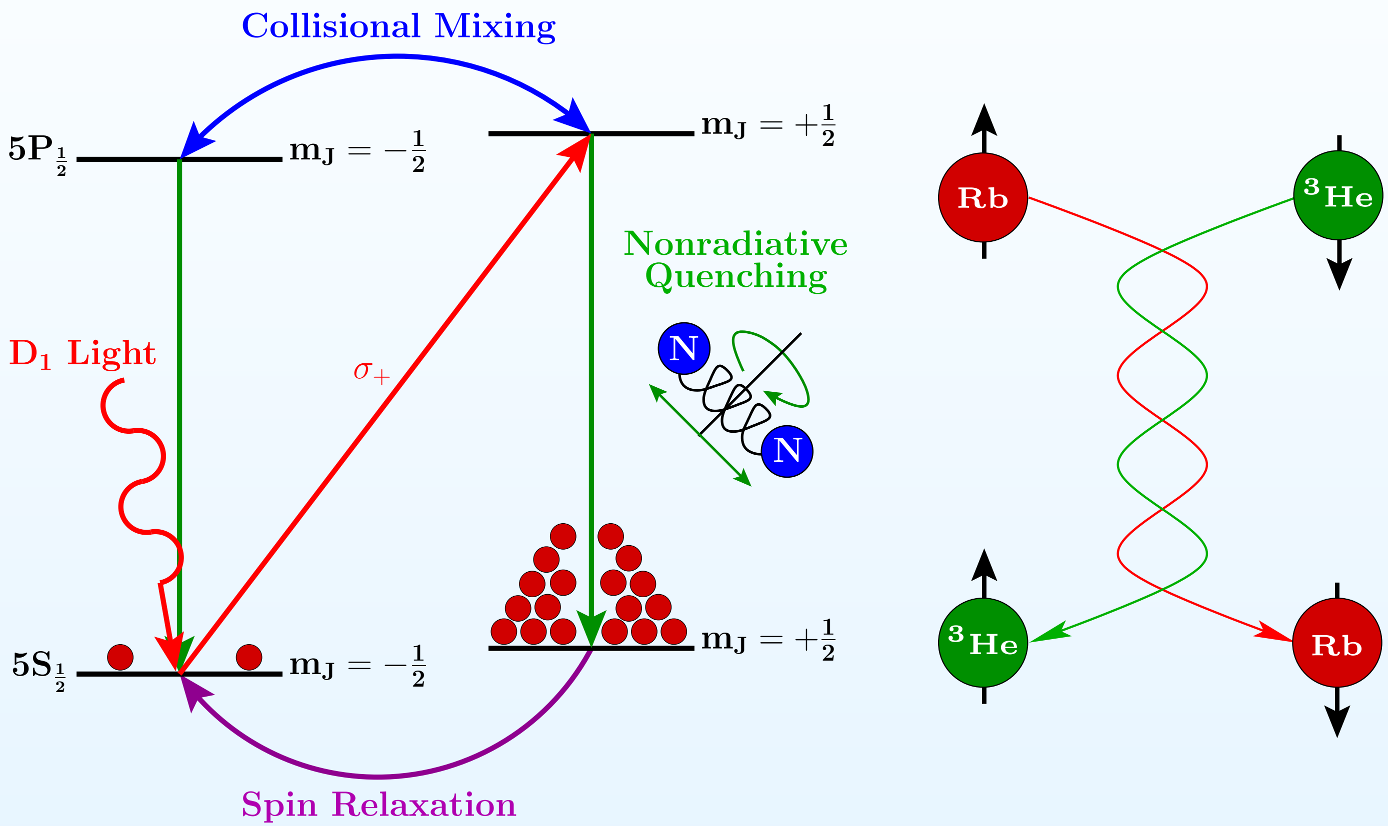


$$A_{\parallel,\perp} = \frac{1}{f_{\rm N_2}P_{\rm tg}P_{\rm beam}} \frac{\frac{N^+}{Q^+LT^+} - \frac{N^-}{Q^-LT^-}}{\frac{N^+}{Q^+LT^+} + \frac{N^-}{Q^-LT^-}}$$

Preliminary Target Polarization



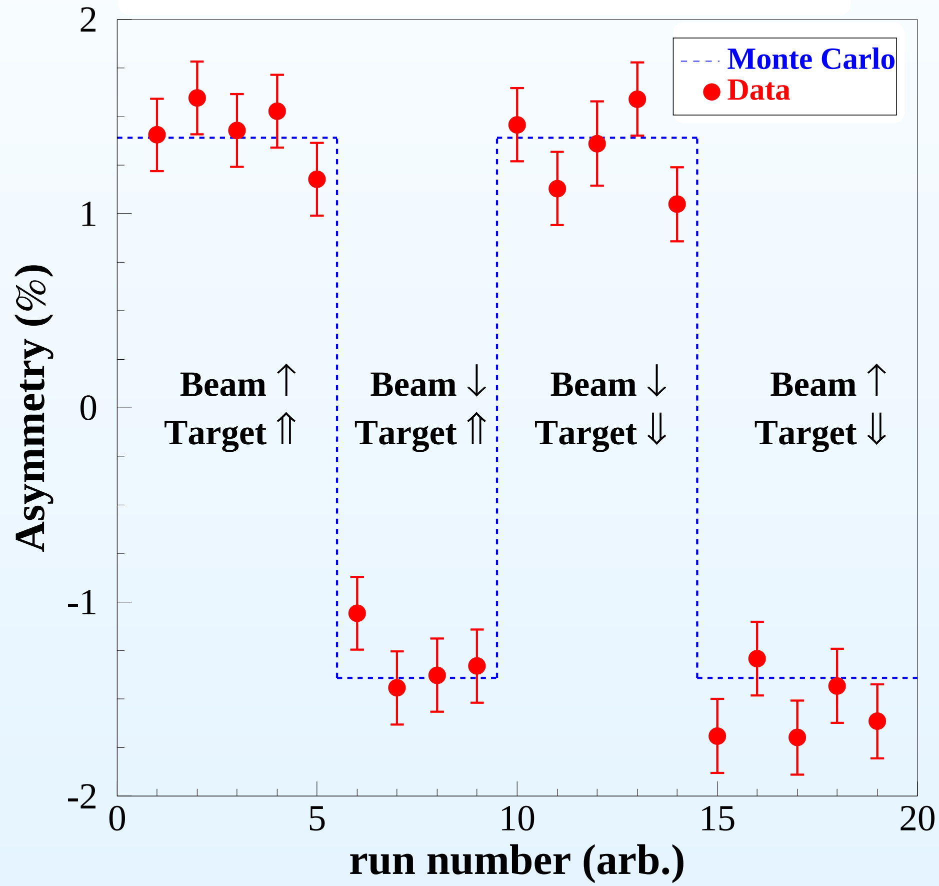
Spin Exchange Optical Pumping



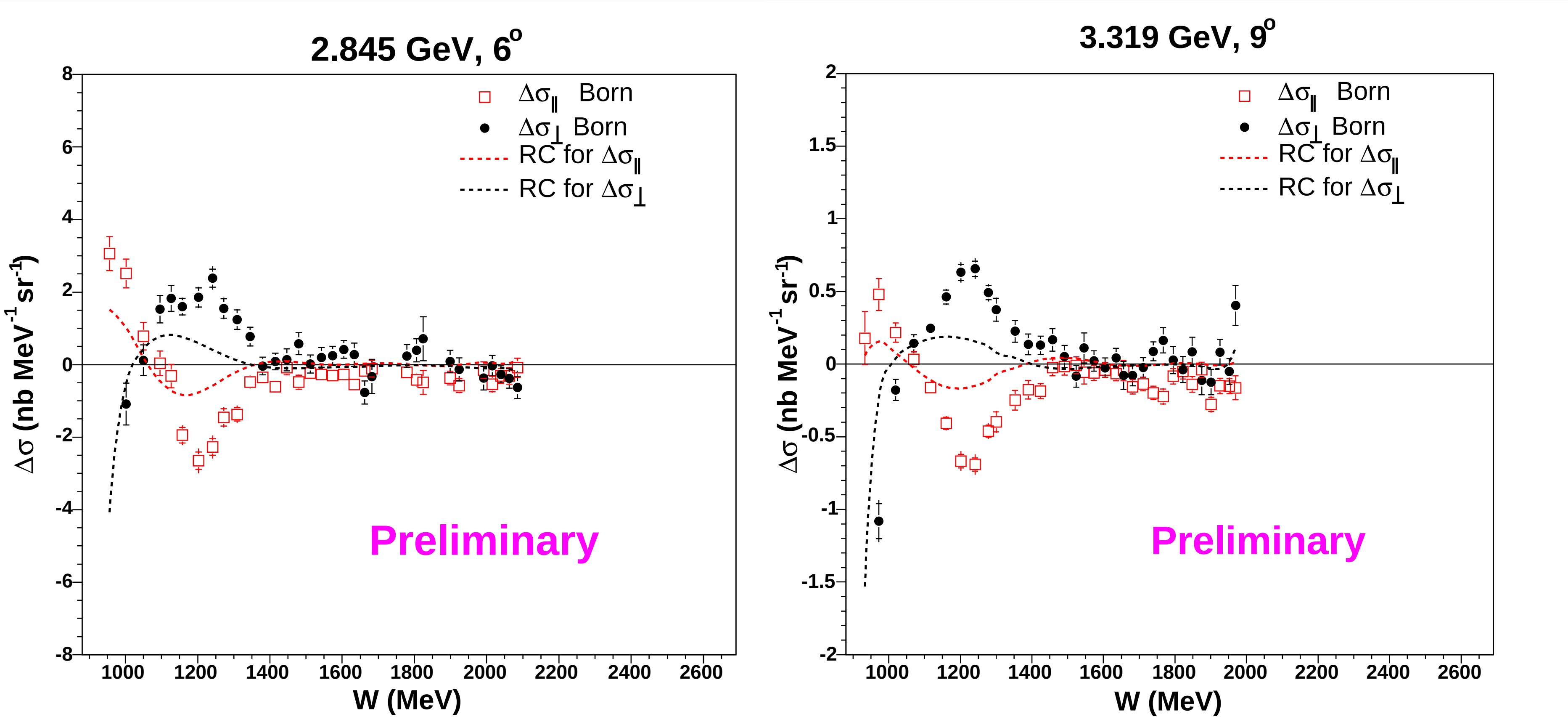
^{3}He nucleus is polarized via spin-exchange with optically pumped Rb atoms.

^{3}He Elastic Asymmetry

- Monte Carlo prediction: 1.390%
- Preliminary data analysis: (1.403 $\pm$ 0.044)% (stat. only) $\chi^2/\mathrm{N_{dof}}$ = 1.08.
- Four target and beam configurations
- For seven out of the twelve beam energies, elastic data were acquired.



Cross Section Differences



Radiative corrections: formalism of L. Mo and Y. Tsai (unpolarized) and POLRAD (polarized), work done by J. Singh.