The Pionless EFT

The Swiss Army Knife of EFTs

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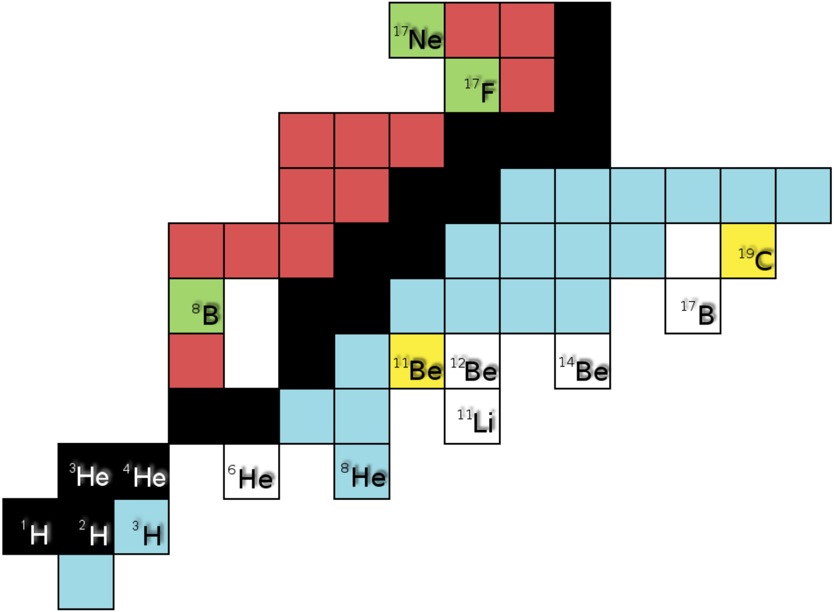
Weakly Bound Nuclei



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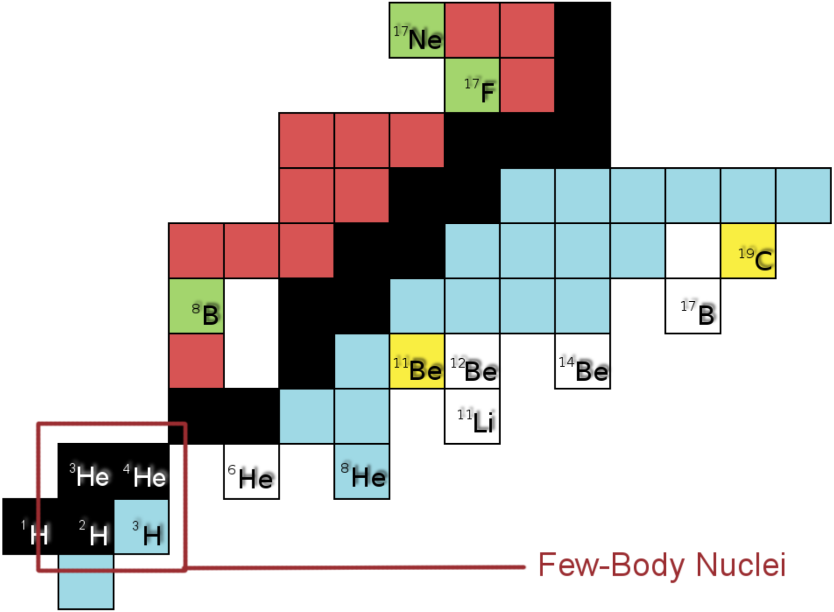

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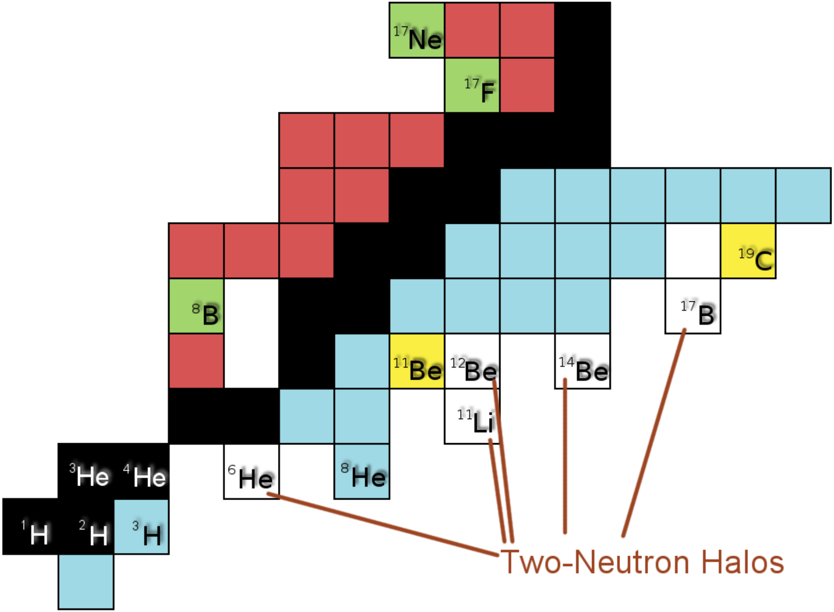

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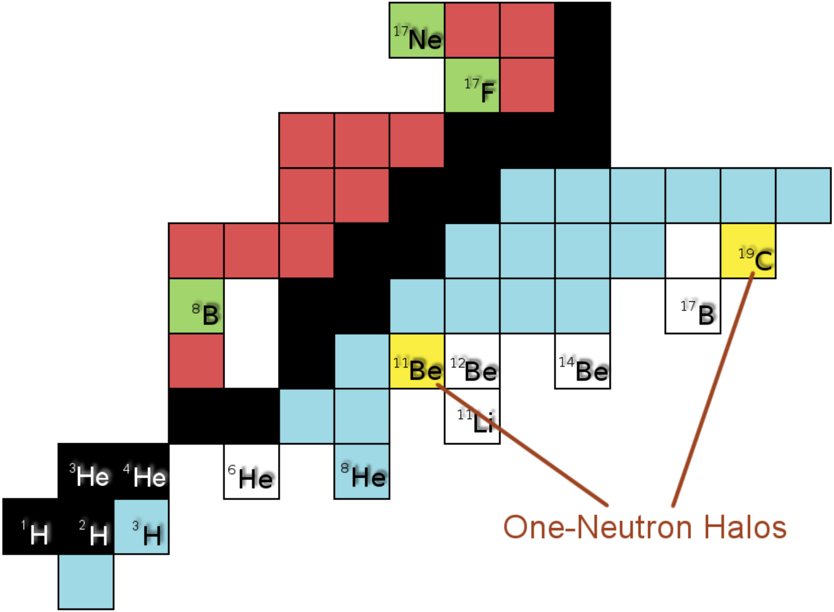

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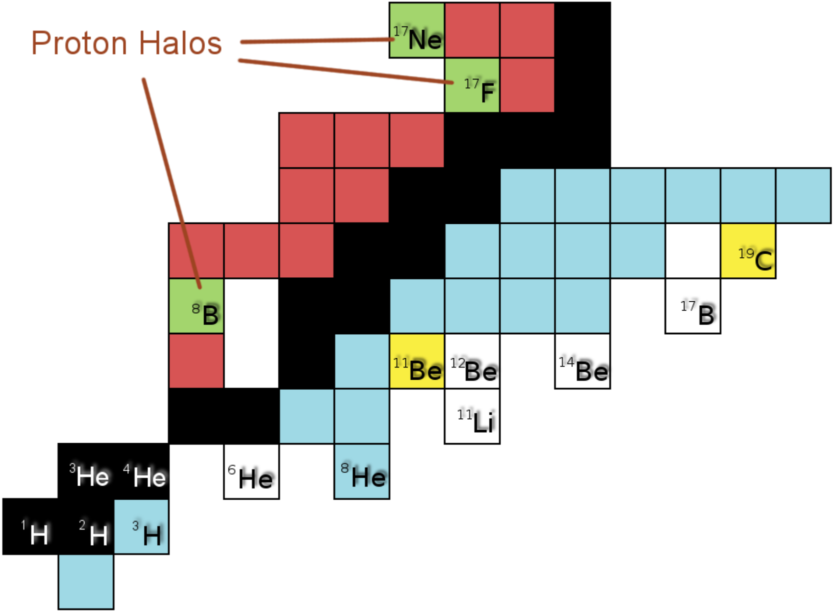

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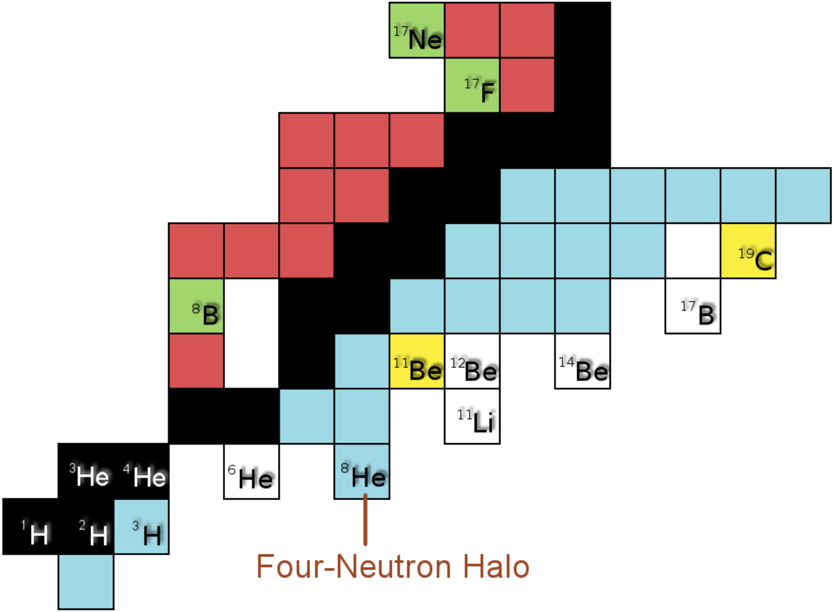

The EFT with Contact Interactions alone

for a finite range potential the t-matrix can be written as

$$t(k) \sim \frac{1}{k\cot\delta - ik}$$

for sufficiently low energies $k\cot\delta$ can be expanded in powers of $k \longrightarrow$ effective range expansion

$$k\cot\delta = -\frac{1}{a} + \frac{r}{2}k^2 + \ldots \ ,$$

or for $a > 0$ expand around the two-body bound state pole $\gamma = \sqrt{MB_2}$

$$k\cot\delta = -\gamma + \frac{r}{2}(\gamma^2 + k^2) + \ldots$$

Consider systems where the scattering length $a \gg \ell$

- such systems have particular universal properties
  → For large positive scattering length we have a bound state at $B_2 \approx \frac{1}{Ma^2}$

  → in the nuclear sector this is the deuteron

  → example in the atomic sector is the ^{4}He dimer

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- separation of scales

  in the nuclear sector:
  - $^1S_0$ $a \sim -24$ fm $\longrightarrow$ $r \sim 3$ fm
  - $^3S_1$ $a \sim 5$ fm $\longrightarrow$ $r \sim 2$ fm

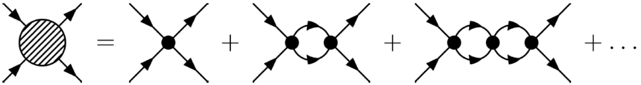
  in the atomic ^{4}He few-body system:
  - $a \sim 100$ Å $\longrightarrow$ $r \sim 10$ Å

In the regime where $k\ell \ll 1$ all interactions look pointlike!

- Use an appropriate EFT (expansion parameters ℓ/a, $k\ell$)
- Most general Lagrangian using only contact interactions:

$$\mathcal{L} = \psi^\dagger \left[i\partial_t + \frac{\overrightarrow{\nabla}^2}{2M} \right] \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 - \frac{D_0}{6} (\psi^\dagger \psi)^3 + \ldots ,$$

- Two-body system (S-waves):



$$\text{Dim. Reg.} \longrightarrow t_{LO} \sim \frac{1}{-1/a + \sqrt{-E - i\epsilon}} \quad \text{w/} \quad C_0 = \frac{4\pi a}{M}$$

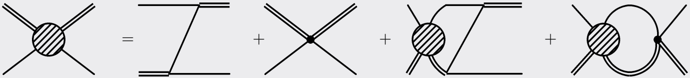
- with correct ordering scheme for diagram topologies (power-counting), this EFT is an expansion in $\ell/|a| \longrightarrow$ suitable for systems with large a

The 2-Body Sector

The most successful calculations in the short-range EFT have been performed in the 2-body sector:

- Form Factors of the Deuteron, Chen et al.
- radiative capture: $n + p \longrightarrow d + \gamma$, Rupak
- muon capture: $\mu^- + d \longrightarrow \nu_\mu + n + n$, Chen et al.
- Deuteron Electro-Disintegration, Christlmeier & Griesshammer
- and many more ...

The Three-Body System



- **integral (STM) equation for atom-dimer scattering:**

$$K(k,p;E) \;=\; \mathcal{Z}(k,p;E) + \int_0^\Lambda \mathrm{d}q''\, q''^2 \mathcal{Z}(k,q'';E)\tau(ME - \frac{3}{4}q''^2)K(q'',p;E)$$

Skorniakov & Ter-Martirosian '56

- **2-body propagator:**

$$\tau(E) = \frac{2}{\pi M^2}\frac{\gamma + \sqrt{-ME}}{E + B_2}$$

- **single nucleon-exchange + 3-body interaction:**

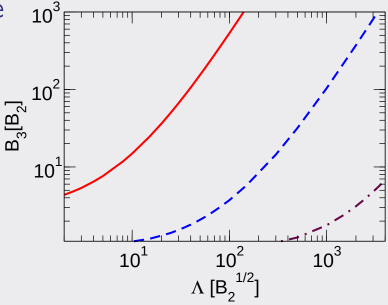
$$\mathcal{Z}(q,q',E) = -\frac{M}{2qq'}\log(\frac{q^2 + qq' + q'^2 - ME}{q^2 - qq' + q'^2 - ME}) + \frac{MH(\Lambda)}{\Lambda^2}$$

Without three-body force

⟶ strong cutoff dependence

⟶ number of bound states increases with cutoff

⟶ relation to Thomas and Efimov effect
$\Rightarrow$ include three-body information



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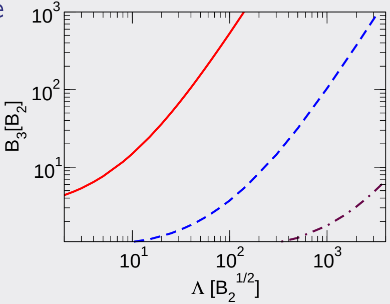


Thus, perform calculations with three-body force:

$\longrightarrow$ use binding energy of weakest three-body state to fix $H(\Lambda)$

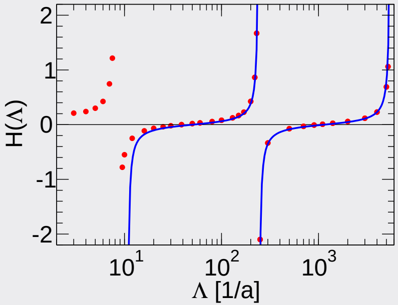
$\longrightarrow$ **this is renormalization**

After Renormalization

⟶ need three-body force for consistent renormalization
(Bedaque, Hammer, van Kolck, PRL 82 (1999) 463)

⟶ three-body system with large scattering length exhibits a **limit cycle**
Wilson, PRD 3 (1971) 1818

Running of $H(\Lambda)$



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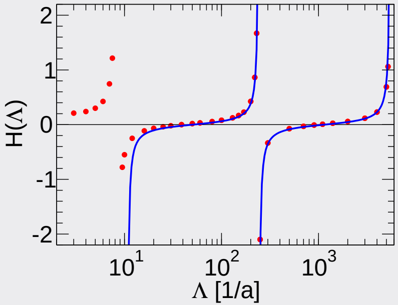


Remember

- We need one three-body observable to fix $H(\Lambda)$

Consequences of the Limit Cycle

The Three-Body parameter

For large Λ the RG-flow of $H(\Lambda)$ is described by:

$$H(\Lambda) = \frac{\sin(s_0 \ln(\Lambda/L_3) - \arctan(1/s_0))}{\sin(s_0 \ln(\Lambda/L_3) + \arctan(1/s_0))} \quad , \text{ with } s_0 \approx 1.0062$$

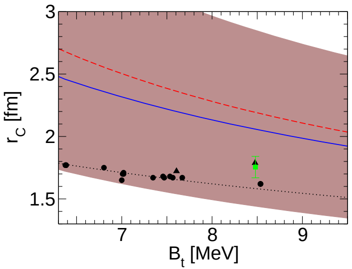
Bedaque, Hammer, van Kolck, PRL 82 (1999) 463

- $H(\Lambda)$ periodic: $\Lambda \rightarrow \Lambda e^{n\pi/s_0} \approx \Lambda(22.7)^n$
- **discrete scale invariance** with consequences for observables, e.g. $B_3^{(m)}/B_3^{(m+1)} \approx 515$
  $\longrightarrow$ this equation holds exactly for all bound states when
  $$\ell \rightarrow 0 \text{ and } a \rightarrow \infty$$
  (Efimov, SJNP 29 (1979) 546)
- **scaling relations** in 3-body observables, e.g. $a_3 - B_3$, $B_3 - r_3$

EM Observables in the 3-Body System

- Keep the scattering length fixed
- Vary one of the three-body observables

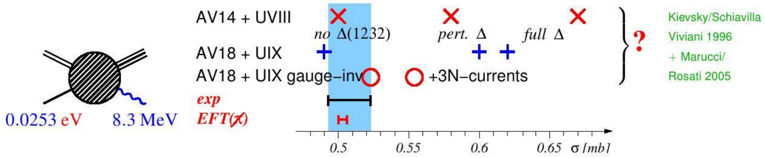
→ See what the others are doing



Hammer, Meissner, LP 2005

- radiative neutron capture at thermal energies

  Sadeghi, Bayegan & Griesshammer 2006



What about a 4-Body force?

RG Analysis:

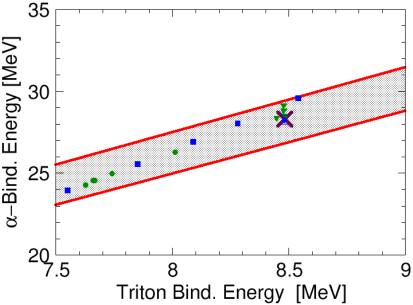
- observables are cutoff independent

$\Longrightarrow$ no 4-body force needed

$\Longrightarrow$ no new parameters in the 4-body sector!
Hammer, Meißner & LP 2004

short-range EFT:

- **describes α-particle**
- **explains Tjon line**
  Hammer, Meißner & LP 2005



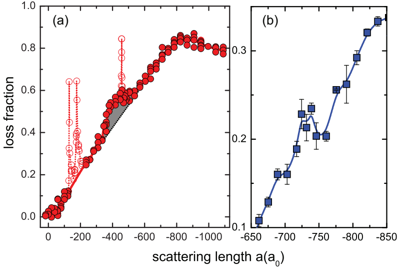
Universal Predictions:

⟹ **2 Tetramer states for every trimer state**
Hammer & LP 2007

- $E_4^0/E_T \sim 5$ and $E_4^1/E_T \sim 1.01$

⟶ confirmed and extended by
Stecher, d'Incao & Greene 2008
  - ⟶ 2 universal tetramer states should be observable in 4-body recombination

- 2 universal tetramer states found experimentally by Innsbruck group
Ferlaino et al. 2009



Beyond Universality

- In the two-body system finite range corrections are easy!
- In the 3-body system higher order 3-body forces enter

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This is relevant:

- For accuracy we need to go beyond leading order! Several applications:
  - Big bang nucleosynthesis, e.g. $p + d \rightarrow {}^3\text{He} + \gamma$
  - α-clusters $\longrightarrow$ ^{12}C Hoyle state
  - Halo nuclei, e.g. ^{6}He

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- So: When does the next 3-body force enter?
- How do we include finite range corrections efficiently and correctly?

Include the Effective Range

Consider the integral equation for particle-dimer scattering:

$$K(k,p;E) \quad = \quad \mathcal{Z}(k,p;E) + \int_0^\Lambda \mathrm{d}q''\, q''^2 \mathcal{Z}(k,q'';E)\tau(ME - \frac{3}{4}q''^2)K(q'',p;E)$$

- Modify the two-body propagator Bedaque *et al* '03

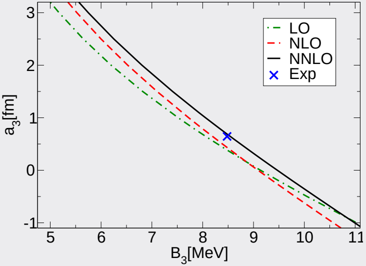
$$\tau^{(n)}(E) = \frac{1}{E+B_2}\frac{2}{\pi M^2}\sum_{i=0}^{n}\left(\frac{r_s}{2}\right)^i [\gamma + \sqrt{-ME}]^{i+1}$$

- At which order does the next three-body force contribute?
  → perturbative analysis gives N2LO **for natural Λ**
  Bedaque et al.2003
  → Renormalization group analysis gives N3LO **for large Λ**
  Phillips & LP 2006
  → full perturbative calculation up to N2LO is on the way

Some Results for the 3-Nucleon System

Range Corrections in the Three-Nucleon System

	a_3 [fm]	B_3 [Mev]
LO	0.65	8.08
NLO	0.65	8.19
NNLO	0.65	8.54
EXP	0.65	8.48



LP, PRC 74 (2006) 037001

- **Note:** Convergence pattern looks strange **but** in fact the NLO correction is actually smaller than expected

NLO in the Unitary Limit

Ji, Phillips & Platter 2008

- in the unitary limit the relevant differential equation is

$$\frac{\hbar^2}{2M}\left(-\frac{\partial^2}{\partial R^2}-\frac{s_0^2+\frac{1}{4}}{R^2}\right)f_0(R)=E\,f_0(R)$$

  which can be solved after renormalizing with a boundary condition or three-body force
- then

$$f_0^{(0)}(R)=\sqrt{R}\,K_{is_0}(\sqrt{2}\kappa R)$$

→ Now we can do perturbation theory on the higher order and analyze the linear range correction to the bound state spectrum in the hyperradial formalism
→**in momentum space for nucleons Hammer & Mehen 2001**

- Obtain the perturbing potential by implementing the NLO Bethe-Peierls condition into the hyperangular equation (Efimov, 1991)

$$V_{\mathrm{NLO}} = -\frac{s_0^2\,\xi_0\,r_s}{R^3} \quad \text{w/} \quad \xi_0 = 0.480$$

compare to Nielsen, Fedorov, Jensen 1998

- We need to renormalize this integral with a three-body force

$$V_{SR}^{(1)}(R) = H_1(\Lambda)\Lambda^2\delta\left(R - \frac{1}{\Lambda}\right)$$

- use H_1 to set the shift for state n_* to 0 by calculating

$$\frac{2M}{\hbar^2}\Delta B_n^{(1)} = s_0^2 r_s \xi_0 \left[\int_{\frac{1}{\Lambda}}^{\infty} dR\, {f_n^{(0)}}^2(R)\frac{1}{R^3} - \frac{2H_1 M}{\hbar^2 s_0^2\, r_s\, \xi_0}\Lambda^2 {f_n^{(0)}}^2\left(\frac{1}{\Lambda}\right)\right]$$

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⋆ linear correction is suppressed due to discrete scale invariance of leading order wave function: $\Delta B_n^{(1)} = 0$ **for all** n

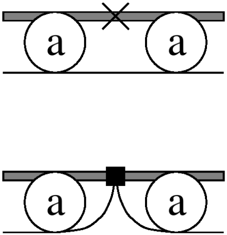
Perturbative Analysis at finite a

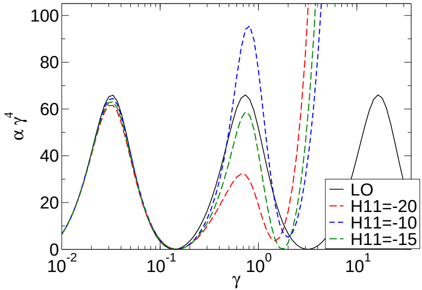
Ji, Phillips & LP *in preparation*

- energy-independent 3-body force H_1
- has form $H_1 = H_{10} + H_{11}/a$

$\longrightarrow$ no new 3-body input for fixed a

$\longrightarrow$ new 3-body input in AMO applications

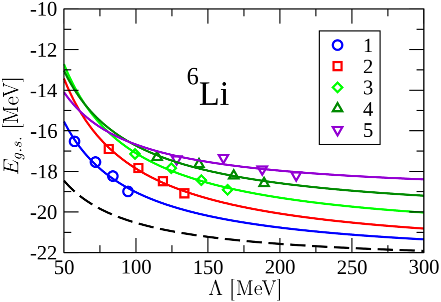




- recombination at NLO, renormalized to 3-body datum in the unitary limit
- reminiscent of quark mass dependence of counterterms

Larger Systems
requires alternative many-body approaches

- Stectu,Barret van Kolck 2006
  - ^{4}He and ^{6}Li at LO
  - $E_{Li6} = 22.6$ MeV
    $E_{Exp} = 31.99$ MeV
  - obtain stable excited ^{4}He state



$\longrightarrow$ higher order corrections have to give large corrections

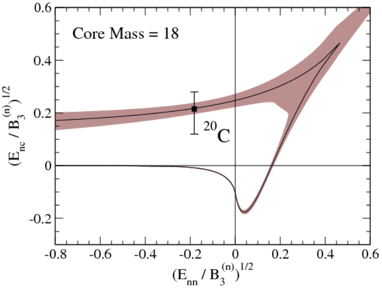
Halo Nuclei

nucleus reducto!

- describe Halos using the minimal set of degrees of freedom: core + nucleons
  - ^{6}He $\longrightarrow \alpha + 2n$ (three-body problem)
  - ^{8}He $\longrightarrow \alpha + 4n$ (five-body problem)
- 2-body: α-n Bertulani, Hammer & van Kolck 2002
  α-α Higa, Hammer & van Kolck 2008

- LO EFT seems to be able to describe Halos well
  Canham & Hammer 2008
  - Binding energies and Radii calculated

  $\longrightarrow$ ^{11}Li, ^{14}Be, ^{12}Be, ^{18}C, ^{20}C



Many-Body Systems

The short-range EFT can also be applied to mayn-body systems:

- natural scattering length: dilute Fermi systems $\longrightarrow$ energy/particle, quasiparticle properties, pairing ...
  Furnstahl & Hammer 2000, Hammer & LP 2002, Furnstahl, Hammer & Puglia 2007
- large scattering length: spin 1/2 fermionic systems (neutron matter) $\longrightarrow$ rederive Tan relations
  Braaten & LP 2008

Summary

- The pionless EFT has been applied successfully to a wide range of observables in atomic, nuclear and particle physics (X3872)
- The pionless EFT is able to describe many well-known scaling properties in few-body systems
  $\longrightarrow$ **all these are a result of a large scattering length in the two-body sector**
- The pionless EFT gives low-energy theorems for few- and many-body observables
- A thorough understanding of higher order corrections is relevant for error estimates and the predictive power of an EFT

Opportunities

Few-Nucleon Sector

- Ay-Problem in Nuclear Physics
- thermal proton capture $p\,d \rightarrow$He$^3\,\gamma$
- Tritium β-decay (relevant for *pp*-fusion)
- 4-body physics relevant to big bang nucleosynthesis

Halo Nuclei:

- Helium-6 awaits
- What about higher order corrections
  $\longrightarrow$ insight into core-nucleon interaction!
- **From Helium-6 to Helium-8!**

α-clusters

- Hoyle state in Carbon-12?