Complex mass renormalization in EFT

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D. Djukanovic, J. Gegelia and S. Scherer, "Mass and width of the Roper resonance using complex-mass renormalization," arXiv:0903.0736 [hep-ph].

Inclusion of heavy baryon resonances such as the Roper in EFT is complicated.

Consistent power counting can be established by using Complex Mass Renormalization scheme – an extension of the on-mass-shell renormalization scheme to unstable particles.

R. G. Stuart, in Z^0 *Physics*, (ed. J. Tran Thanh Van (Editions Frontieres, Gif-sur-Yvette, 1990), p.41.)

A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, Nucl. Phys. **B560**, 33 (1999).

J. Gasser and H. Leutwyler, "Quark Masses," Phys. Rept. **87**, 77 (1982).

" In principle, the renormalization program is straightforward: one calculates quantities of physical interest in terms of the bare parameters at given, large value of (ultraviolet cutoff) Λ. Once a sufficient number of physical quantities have been determined as functions of the bare parameters one inverts the result and expresses the bare parameters in terms of physical quantities, always working at some given, large value of Λ. Finally, one uses these expressions to eliminate the bare parameters in all other quantities of physical interest. Renormalizability guarantees that this operation at the same time also eliminates the cutoff."

Effective Lagrangian:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\pi^{(2)} + \mathcal{L}_R + \mathcal{L}_{NR} + \mathcal{L}_{\Delta R}\,,$$

where

$$\begin{aligned}\mathcal{L}_0 \;=\;& \bar{N}\,(i\not{D} - m_{N0})N + \bar{R}(i\not{D} - m_{R0})R \\ & -\bar{\Psi}_\mu \xi^{\frac{3}{2}}[(i\not{D} - m_{\Delta 0})\,g^{\mu\nu} - i\,(\gamma^\mu D^\nu + \gamma^\nu D^\mu) \\ & +i\,\gamma^\mu \not{D}\gamma^\nu + m_{\Delta 0}\,\gamma^\mu\gamma^\nu]\xi^{\frac{3}{2}}\Psi_\nu.\end{aligned}$$

N, R and Ψ_ν - nucleon, Roper and Δ with bare masses m_{N0}, m_{R0} and $m_{\Delta 0}$. $\xi^{\frac{3}{2}}$ - isospin projector.

The lowest-order Goldstone boson Lagrangian

$$\mathcal{L}_\pi^{(2)} = \frac{F^2}{4}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{F^2 M^2}{4}\mathrm{Tr}\left(U^\dagger + U\right) ,$$

where the pions are contained in (2×2) matrix U. F denotes the pion-decay constant in the chiral limit: $F_\pi = F[1+\mathcal{O}(q^2)] = 92.4$ MeV; M is the pion mass at leading order in the quark-mass expansion.

Interaction terms $\mathcal{L}_R$, $\mathcal{L}_{NR}$, and $\mathcal{L}_{\Delta R}$ following

B. Borasoy, P. C. Bruns, U.-G. Meißner, and R. Lewis, Phys. Lett. B **641**, 294 (2006):

Leading order pion-Roper coupling

$$\mathcal{L}_R^{(1)} = \frac{g_R}{2}\,\bar{R}\gamma^\mu\gamma_5 u_\mu R\,,$$

g_R is an unknown coupling constant, $u = \sqrt{U}$ and

$$u_\mu = i\left[u^\dagger\partial_\mu u - u\partial_\mu u^\dagger\right].$$

Next-to-leading-order Roper Lagrangian

$$\mathcal{L}_R^{(2)} = c_{1,0}^*\langle\chi_+\rangle\,\bar{R}\,R + \cdots\,,$$

where $c_{1,0}^*$ is an unknown bare coupling constant and $\chi_+ = M^2(U + U^\dagger)$.

Interaction between the nucleon and the Roper

$$\mathcal{L}_{NR}^{(1)} = \frac{g_{NR}}{2}\,\bar{R}\gamma^\mu\gamma_5 u_\mu N + \text{h.c.}$$

with an unknown coupling constant g_{NR}.

Leading-order interaction between delta and Roper

$$\mathcal{L}_{\Delta R}^{(1)} = -g_{\Delta R}\,\bar{\Psi}_\mu\,\xi^{\frac{3}{2}}\,(g^{\mu\nu} + \tilde{z}\,\gamma^\mu\gamma^\nu)\,u_\nu\,R + \text{h.c.}\,,$$

$g_{\Delta R}$ - unknown coupling constant.

Complex-mass renormalization: Split the bare parameters into renormalized parameters and counterterms and choose the renormalized masses as the poles of the dressed propagators in the chiral limit:

$$\begin{aligned} m_{R0} &= z_\chi + \delta z_\chi\,, \\ m_{N0} &= m + \delta m\,, \\ m_{\Delta 0} &= z_{\Delta\chi} + \delta z_{\Delta\chi}\,, \\ c_{1,0}^* &= c_1^* + \delta c_1^*\,, \\ &\cdots\,. \end{aligned}$$

Power counting:

vertex obtained from an $\mathcal{O}(q^n)$ Lagrangian $\sim q^n$,

pion propagator $\sim q^{-2}$,

nucleon propagator $\sim q^{-1}$,

integration of a loop $\sim q^4$.

Δ and Roper propagators $\sim q^{-1}$.

Dressed propagator of the Roper

$$iS_R(p) = \frac{i}{\not{p} - z_\chi - \Sigma_R(\not{p})},$$

where $\Sigma_R(\not{p})$ is the self-energy of the Roper.

Pole of the dressed propagator S_R

$$z - z_\chi - \Sigma_R(z) = 0.$$

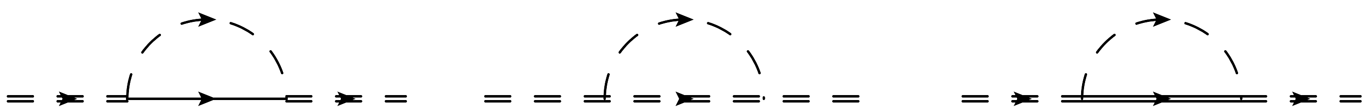
Define the pole mass and the width:

$$z = m_R - i\,\frac{\Gamma_R}{2}.$$

To order $\mathcal{O}(q^3)$ self-energy consists of tree-contribution

$$\Sigma_{\text{tree}} = 4\, c_1^* M^2 \,,$$

and the loop diagrams:



(a) (b) (c)

$$
\begin{aligned}
\Sigma_{(a)} &= \frac{3g_{NR}^2}{128\pi^2F^2}[\hat{O}_1(m)A_0\left(m^2\right)+\hat{O}_2(m)A_0\left(M^2\right)\\
&+\hat{O}_3(m)B_0\left(p^2,m^2,M^2\right)],\\
\Sigma_{(b)} &= \frac{3g_{R}^2}{128\pi^2F^2}[\hat{O}_1(z_\chi)A_0\left(z_\chi^2\right)+\hat{O}_2(z_\chi)A_0\left(M^2\right)\\
&+\hat{O}_3(z_\chi)B_0\left(p^2,z_\chi^2,M^2\right)],\\
\Sigma_{(c)} &= \frac{g_{\Delta R}^2}{48\pi^2F^2}[\hat{O}_4+\hat{O}_5A_0\left(z_{\Delta\chi}^2\right)+\hat{O}_6A_0\left(M^2\right)\\
&+\hat{O}_7B_0\left(p^2,z_{\Delta\chi}^2,M^2\right)],
\end{aligned}
$$

where

$$
\begin{aligned}
\widehat{O}_1(x) &= \not{p}\left(1+\frac{x^2}{p^2}\right)+2x, \\
\widehat{O}_2(x) &= \not{p}\left(1-\frac{x^2}{p^2}\right), \\
\widehat{O}_3(x) &= \not{p}\left[-p^2\left(1-\frac{x^2}{p^2}\right)^2+M^2\left(1+\frac{x^2}{p^2}\right)\right]+2M^2x. \\
\widehat{O}_4 &= \frac{1}{6}[3\not{p}z_{\Delta\chi}^2-12p^2z_{\Delta\chi}-4\not{p}p^2+4p^2\frac{p^2-3M^2}{z_{\Delta\chi}} \\
&\quad +\not{p}\frac{2(p^2)^2-3M^4-8p^2M^2}{z_{\Delta\chi}^2}],
\end{aligned}
$$

$$\hat{O}_5 = \frac{1}{p^2}[\not{p}\, z^2_{\Delta\chi} + 2p^2 z_{\Delta\chi} - \not{p}\left(2M^2 + p^2\right) + 2p^2 \frac{p^2 - M^2}{z_{\Delta\chi}} + \not{p}\, \frac{\left(M^2 - p^2\right)^2}{z^2_{\Delta\chi}}],$$

$$\hat{O}_6 = -\frac{1}{p^2}[\not{p}\, z^2_{\Delta\chi} + 2p^2 z_{\Delta\chi} - 2M^2 \not{p} - 2p^2 \frac{M^2 + p^2}{z_{\Delta\chi}} + \not{p}\, \frac{M^4 - 3p^2 M^2 - (p^2)^2}{z^2_{\Delta\chi}}],$$

$$\hat{O}_7 = -\frac{1}{p^2}\left[\not{p}\, z^2_{\Delta\chi} + 2p^2 z_{\Delta\chi} + \not{p}\left(p^2 - M^2\right)\right] \times \left[z^2_{\Delta\chi} - 2\left(M^2 + p^2\right) + \frac{\left(M^2 - p^2\right)^2}{z^2_{\Delta\chi}}\right].$$

Loop functions are given as

$$
\begin{aligned}
A_0\left(m^2\right) &= -32\pi^2\lambda\, m^2 - 2\, m^2 \ln\frac{m}{\mu}\,, \\
B_0\left(p^2, m_1^2, m_2^2\right) &= -32\pi^2\lambda + 2\ln\frac{\mu}{m_2} - 1 - \frac{1}{2}\left(1 + \frac{m_2^2}{m_1^2(\omega-1)}\right) \\
&\quad \times {}_2F_1\left(1,2;3;1+\frac{m_2^2}{m_1^2(\omega-1)}\right) \\
&\quad -\frac{\omega}{2}\ {}_2F_1\left(1,2;3;\omega\right)\,, \\
\omega &= \frac{m_1^2 - m_2^2 + p^2 + \sqrt{\left(m_1^2 - m_2^2 + p^2\right)^2 - 4m_1^2p^2}}{2m_1^2}\,,
\end{aligned}
$$

${}_2F_1\left(a,b;c;z\right)$ is the hypergeometric function and

$$
\lambda = \frac{1}{16\,\pi^2}\left\{\frac{1}{n-4} - \frac{1}{2}\left[\ln(4\pi) + \Gamma'(1) + 1\right]\right\}\,.
$$

$B_0\left(p^2, m_1^2, m_2^2\right)$ is a single-valued function of ω.

As a function of p^2 it is two-valued and has two branch points $p^2 = (m_1 - m_2)^2$ and $p^2 = (m_1 + m_2)^2$.

Branch points of the self-energy diagrams (b) and (c) are complex, $z_\chi \pm M$ and $z_{\Delta\chi} \pm M$.

Branch points of the exact two-point functions are determined by poles of the fully dressed propagators of the Roper and Δ.

To implement the complex-mass renormalization scheme, we expand the self-energy loop diagrams in powers of M, $\not{p} - z_\chi$, and $p^2 - z_\chi^2$, which all count as $\mathcal{O}(q)$. We subtract those terms which violate the power counting.

Subtraction terms evaluated at $\not{p} = z_\chi$:

$$
\begin{aligned}
\Sigma^{\mathrm{ST}}_{(a)} &= -\frac{3g_{NR}^2(m+z_\chi)^2}{128\pi^2F^2z_\chi}\left[(m-z_\chi)^2B_0\left(z_\chi^2,0,m^2\right)-A_0\left(m^2\right)\right] \\
&\quad +\frac{3g_{NR}^2(m+z_\chi)M^2}{64\pi^2F^2z_\chi^3}\left[-2m^3\ln\left(\frac{m}{\mu}\right)-i\pi m^3+z_\chi^2m\right. \\
&\quad \left.-32\pi^2z_\chi^3\lambda+\left(m^3-z_\chi^3\right)\ln\left(\frac{z_\chi^2-m^2}{\mu^2}\right)+i\pi z_\chi^3\right], \\
\Sigma^{\mathrm{ST}}_{(b)} &= \frac{3g_R^2z_\chi}{32\pi^2F^2}\,A_0\left(z_\chi^2\right)-\frac{3g_R^2z_\chi M^2}{32\pi^2F^2}\left[32\pi^2\lambda+2\ln\left(\frac{z_\chi}{\mu}\right)-1\right],
\end{aligned}
$$

$$
\begin{aligned}
\Sigma^{\mathrm{ST}}_{(c)} \;=\;& -\frac{g^2_{\Delta R}}{288F^2\pi^2 z^2_{\Delta\chi} z_\chi}\Big[6(z_{\Delta\chi}-z_\chi)^2(z_{\Delta\chi}+z_\chi)^4 B_0\left(z^2_\chi, 0, z^2_{\Delta\chi}\right) \\
& + z^2_\chi(-3z^4_{\Delta\chi} + 12 z_\chi z^3_{\Delta\chi} + 4z^2_\chi z^2_{\Delta\chi} - 4z^3_\chi z_{\Delta\chi} - 2z^4_\chi) \\
& -6\left(z^4_{\Delta\chi} + 2z_\chi z^3_{\Delta\chi} - z^2_\chi z^2_{\Delta\chi} + 2z^3_\chi z_{\Delta\chi} + z^4_\chi\right) A_0\left(z^2_{\Delta\chi}\right)\Big] \\
& + \frac{g^2_{\Delta R} M^2}{72\pi^2 F^2 z^2_{\Delta\chi} z^3_\chi}[-6i\pi z^6_{\Delta\chi} - 6(2z_{\Delta\chi}+3z_\chi) z^5_{\Delta\chi} \ln\left(\frac{z_{\Delta\chi}}{\mu}\right) \\
& -9i\pi z_\chi z^5_{\Delta\chi} + 6z^2_\chi z^4_{\Delta\chi} + 9z^3_\chi z^3_{\Delta\chi} + 3z^4_\chi z^2_{\Delta\chi} - 288\pi^2\lambda z^5_\chi z_{\Delta\chi} \\
& +9i\pi z^5_\chi z_{\Delta\chi} + z^6_\chi - 192\pi^2\lambda z^6_\chi \\
& + \left(6z^6_{\Delta\chi} + 9z_\chi z^5_{\Delta\chi} - 9z^5_\chi z_{\Delta\chi} - 6z^6_\chi\right)\ln\left(\frac{z^2_\chi - z^2_{\Delta\chi}}{\mu^2}\right) \\
& +6i\pi z^6_\chi]. \qquad (1)
\end{aligned}
$$

The above expressions are exactly canceled by counterterm contributions generated by δz_χ and δc_1^*.

The pole of the Roper propagator:

$$z = z_\chi - 4\, c_1^* M^2 + \left[\Sigma_{(a)} + \Sigma_{(b)} + \Sigma_{(c)}\right]_{\not p = z_\chi} - \Sigma^{\mathrm{ST}}_{(a)} - \Sigma^{\mathrm{ST}}_{(b)} - \Sigma^{\mathrm{ST}}_{(c)}\,.$$

Loop contribution in z satisfies the power counting, i.e. is of $\mathcal{O}(q^3)$.

Non-analytic (in pion mass) terms agree with

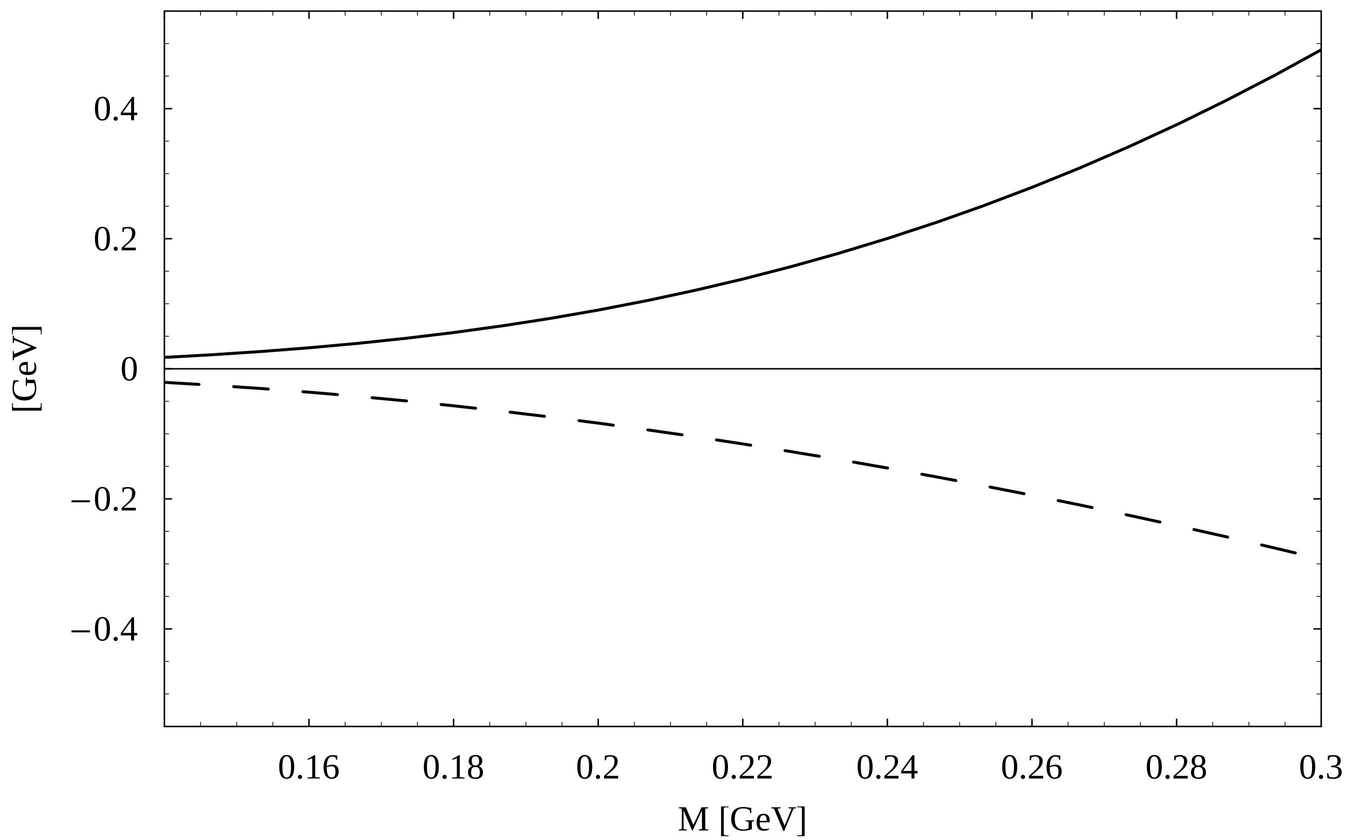
B. Borasoy, P. C. Bruns, U.-G. Meißner, and R. Lewis, Phys. Lett. B **641**, 294 (2006):

Our result is a closed expression and need not be expanded.

Numerical estimate: $F = 0.092$ GeV, $M = 0.140$ GeV, $m = 0.940$ GeV, $z_{\Delta\chi} = (1.210 - 0.100\, i/2)$ GeV, $z_\chi = (1.365 - 0.190\, i/2)$ GeV, $\mu = 1$ GeV, $g_R = 1$, $g_{\Delta R} = 1$, $g_{NR} = 0.45$

$$z = \left[\left(1.365 - \frac{i}{2} 0.190 \right) - 0.0784\, c_1^* + \left(0.0175 - \frac{i}{2} 0.042 \right) \right] \text{GeV}\,.$$

Contributions of loop diagrams to the Roper pole as functions of the pion mass M.



Summary

- We considered the chiral corrections to the mass and the width of the Roper resonance in the framework of the low-energy EFT of QCD.

- Complex-mass renormalization scheme has been applied.

- Mass and the width of the Roper in the chiral limit are input parameters in our approach.

- Chiral corrections to the Mass and the width have been calculated in a systematic way.