Mesons and glueballs in chiral approach and AdS/QCD

Valery Lyubovitskij

Institut für Theoretische Physik, Universität Tübingen

Kepler Center for Astro and Particle Physics, Germany

EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

in collaboration with

Thomas Gutsche
Amand Faessler
Tanja Branz
Franceso Giacosa
Ivan Schmidt
Malte Tichy
Alfredo Vega

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Objectives of the talk

- Chiral approach: Decay properties of S, PS, V and T mesons above 1 GeV including their mixing with glueballs

- AdS/QCD: Mass spectrum and decay properties of $q\bar{q}$, $Q\bar{q}$ and $Q\bar{Q}$ mesons, exotic hadrons $(\bar{q}q)^m G^n$

Based on papers

PRD 79 (2009) 014036; PRD 72 (2005) 094006, 114021; PLB 622 (2005) 277; arXiv:0904.3414; 0906.1220 [hep-ph]

Mesons and Glueballs in chiral approach

- Effective chiral Lagrangians – phenomenological tool to extract possible glueball-quarkonia mixing scenarios from observed decays
- Scalar mesons above 1 GeV $a_0(1450)$, $K_0^*(1430)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$

$$\begin{aligned}
\mathcal{L}_{\rm eff} &= \frac{F^2}{4}\left\langle D_\mu U\, D^\mu U^\dagger + \chi_+ \right\rangle + \frac{1}{2}\left\langle D_\mu \mathcal{S} D^\mu \mathcal{S} - M_{\mathcal{S}}^2 \mathcal{S}^2 \right\rangle + \frac{1}{2}(\partial_\mu G \partial^\mu G - M_G^2 G^2) \\
&+ c_d^s \left\langle \mathcal{S}\, u_\mu\, u^\mu \right\rangle + c_m^s \left\langle \mathcal{S}\, \chi_+ \right\rangle + \frac{c_d^g}{\sqrt{3}} G \left\langle u_\mu\, u^\mu \right\rangle + \frac{c_m^g}{\sqrt{3}} G \left\langle \chi_+ \right\rangle \\
&+ c_e^s \left\langle \mathcal{S}\, F_{\mu\nu}^+\, F^{+\,\mu\nu} \right\rangle + \frac{c_e^g}{\sqrt{3}} G \left\langle F_{\mu\nu}^+\, F^{+\,\mu\nu} \right\rangle + \mathcal{L}_{\rm mix}^P + \mathcal{L}_{\rm mix}^S
\end{aligned}$$

$$\mathcal{L}_{\rm mix}^S = e_m^S \left\langle \mathcal{S}^2 \chi_+ \right\rangle + k_m^S S_0 \left\langle \chi_+ \right\rangle - \gamma_{S_0} \frac{M_{S_0}^2}{2} S_0^2 - \gamma_{S_8} \frac{M_{S_8}^2}{2} S_8^2 - \sqrt{3} f G S_0$$

$$\begin{aligned}
\mathcal{L}^S &= -\frac{1}{2}\vec{a}_0(\Box + M_{a_0}^2)\vec{a}_0 - K_0^{*\,\dagger}(\Box + M_{K_0^*}^2)\, K_0^* - \frac{1}{2} G(\Box + M_G^2) G \\
&- \frac{1}{2} M_{S_8}^2 S_8^2 - \frac{1}{2} M_{S_0}^2 S_0^2 - z_S S_0 S_8 - \sqrt{3} f G S_0
\end{aligned}$$

Mesons and Glueballs in chiral approach

- Masses of scalar multiplet

$$
\begin{aligned}
M_{a_0}^2 &= M_{\mathcal{S}}^2 - 4e_m^S M_\pi^2, \quad M_{K_0^*}^2 = M_{\mathcal{S}}^2 - 4e_m^S M_K^2 \\
M_{S_8}^2 &= M_{\mathcal{S}}^2(1+\gamma_{S_8}) - \frac{4}{3}e_m^S(4M_K^2 - M_\pi^2), \quad M_{S_0}^2 = M_{\mathcal{S}}^2(1+\gamma_{S_0}) - \frac{4}{3}e_m^S(2M_K^2 + M_\pi^2) \\
z_S &= \frac{8\sqrt{2}}{3}\left(e_m^S + \frac{\sqrt{3}}{2}k_m^S\right)(M_K^2 - M_\pi^2)
\end{aligned}
$$

- Scalar-isoscalar part of the Lagrangian involving the fields N, S and G

$$
\tilde{\mathcal{L}}^S = -\frac{1}{2}\,\Phi\,(\Box + M_\Phi^2)\,\Phi\,, \qquad \Phi = \begin{pmatrix} N \\ G \\ S \end{pmatrix}, \qquad M_\Phi^2 = \begin{pmatrix} M_N^2 & \sqrt{2}f & \varepsilon \\ \sqrt{2}f & M_G^2 & f \\ \varepsilon & f & M_S^2 \end{pmatrix}.
$$

- The bare masses M_N, M_S and flavor mixing parameter ε are:

$$
\begin{aligned}
M_N^2 &= \frac{2}{3}M_{S_0}^2 + \frac{1}{3}M_{S_8}^2 + \frac{2\sqrt{2}}{3}z_S\,, \qquad M_S^2 = \frac{1}{3}M_{S_0}^2 + \frac{2}{3}M_{S_8}^2 - \frac{2\sqrt{2}}{3}z_S\,, \\
\varepsilon &= \frac{\sqrt{2}}{3}\left(M_{S_0}^2 - M_{S_8}^2\right) - \frac{1}{3}z_S\,.
\end{aligned}
$$

Mesons and Glueballs in chiral approach

- Physical states

$$\begin{pmatrix} f_1 \equiv f_0(1370) \\ f_2 \equiv f_0(1500) \\ f_3 \equiv f_0(1710) \end{pmatrix} = B \begin{pmatrix} N \\ G \\ S \end{pmatrix} .$$

- B diagonalizes mass mixing matrix M_Φ^2 as

$$B\ M_\Phi^2\ B^T = M_f^2 = \mathrm{diag}\Big\{ M_{f_1}^2, M_{f_2}^2, M_{f_3}^2 \Big\}$$

- Parameters: $\underbrace{M_N,\ M_G,\ M_S}_{\text{bare masses}}$ $\underbrace{f,\ \varepsilon}_{\text{mixing parameters}}$ $\underbrace{c_d^s,\ c_m^s,\ c_d^g,\ c_m^g}_{\text{decay parameters}}$

Mesons and Glueballs in chiral approach

- Without direct glueball decay to mesons: $c_d^g = c_m^g = 0$

$$B_I = \begin{pmatrix} 0.86 & 0.45 & 0.24 \\ -0.45 & 0.89 & -0.06 \\ -0.24 & -0.06 & 0.97 \end{pmatrix}, \qquad B_{II} = \begin{pmatrix} 0.81 & 0.54 & 0.19 \\ -0.49 & 0.49 & 0.72 \\ -0.30 & -0.68 & 0.67 \end{pmatrix}$$

- Including direct glueball decay to mesons: $c_d^g \sim 1 - 2$ MeV, $c_m^g \sim 25$ MeV

$$B_{III} = \begin{pmatrix} 0.79 & 0.56 & 0.26 \\ -0.58 & 0.81 & 0.02 \\ -0.20 & -0.16 & 0.97 \end{pmatrix}, \qquad B_{IV} = \begin{pmatrix} 0.82 & -0.07 & 0.57 \\ -0.57 & \sim 0 & 0.82 \\ -0.06 & 0.99 & 0.04 \end{pmatrix}$$

- Although many experimental results can be reproduced, and the presence of a scalar glueball and its mixing with scalar quarkonia explains many features of the scalar meson spectroscopy, further work, both theoretically and experimentally, is needed to rule out some mixing scenarios in favour of others.

Mesons and Glueballs in chiral approach

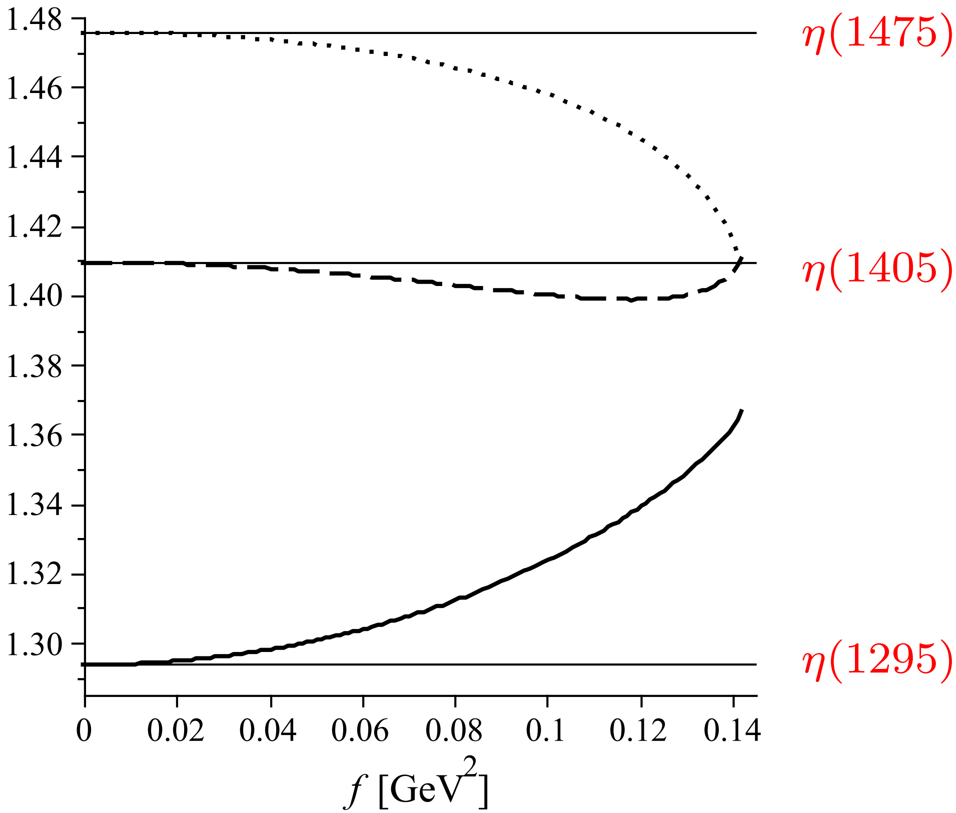
- Parameters: $M_N = 1.455$ GeV, $M_G = 1.490$ GeV, $M_S = 1.697$ GeV, $f = 0.065$ GeV^2, $\varepsilon = 0.211$ GeV^2, $c_d^s = 8.48$ MeV, $c_m^s = 2.59$ MeV.
- Fitted mass and decay properties of scalar mesons

Quantity	Exp	Theory	χ_i^2
M_{f_1} (MeV)	1350 ± 150	1417	0.202
M_{f_2} (MeV)	1507 ± 5	1507	~ 0
M_{f_3} (MeV)	1714 ± 5	1714	0.003
$\Gamma_{f_2\to\pi\pi}$ (MeV)	38.0 ± 4.6	38.52	0.011
$\Gamma_{f_2\to\overline{K}K}$ (MeV)	9.4 ± 1.7	10.36	0.322
$\Gamma_{f_2\to\eta\eta}$ (MeV)	5.6 ± 1.3	1.90	8.109
$\Gamma_{f_3\to\pi\pi}/\Gamma_{f_3\to\overline{K}K}$	0.20 ± 0.06	0.212	0.036
$\Gamma_{f_3\to\eta\eta}/\Gamma_{f_3\to\overline{K}K}$	0.48 ± 0.15	0.249	2.446
$\Gamma_{a_0\to\overline{K}K}/\Gamma_{a_0\to\pi\eta}$	0.88 ± 0.23	0.838	0.032
$\Gamma_{a_0\to\pi\eta'}/\Gamma_{a_0\to\pi\eta}$	0.35 ± 0.16	0.288	0.150
$\Gamma_{K_0^*\to K\pi}$ (MeV)	273 ± 51	59.10	17.590
$(\Gamma_{f_3})_{2P}$ (MeV)	140 ± 10	143.27	0.110
χ^2_{tot}	-	-	29.01

Mesons and Glueballs in chiral approach

- Excited PS mesons: $\pi(1300)$, $K(1460)$, $\eta(1295)$, $\eta(1405)$, $\eta(1475)$

$$\begin{pmatrix} \eta_1 \equiv \eta(1295) \\ \eta_2 \equiv \eta(1405) \\ \eta_3 \equiv \eta(1475) \end{pmatrix} = B \begin{pmatrix} N \\ G \\ S \end{pmatrix} , \qquad M_\Phi^2 = \begin{pmatrix} M_N^2 & \sqrt{2} f & 0 \\ \sqrt{2} f & M_G^2 & f \\ 0 & f & M_S^2 \end{pmatrix}$$



Mesons and Glueballs in chiral approach

- Tensor mesons: $a_2(1320)$, $K_2^*(1430)$, $f_2(1270)$, $f_2'(1525)$
- Parameters: $c^8_{TPP} = 0.0353$ GeV, $c^0_T = 0.0410$ GeV, $\theta_T = 28.78°$
- $f_2 = T^0 \cos\theta_T + T^8 \sin\theta_T$, $f_2' = -T^0 \sin\theta_T + T^8 \cos\theta_T$
- Decay properties of tensor mesons

Mode	Exp (MeV)	Th. (MeV)	χ_i^2
$\Gamma_{f_2\to\pi\pi}$	157.0 ± 7.6	153.51	0.210
$\Gamma_{f_2\to\bar{K}K}$	8.5 ± 0.9	9.15	0.526
$\Gamma_{f_2\to\eta\eta}$	0.83 ± 0.20	0.80	0.023
$\Gamma_{f_2'\to\pi\pi}$	0.60 ± 0.16	0.55	0.102
$\Gamma_{f_2'\to\bar{K}K}$	64.8 ± 7.6	41.64	9.288
$\Gamma_{f_2'\to\eta\eta}$	7.5 ± 2.9	6.49	0.196
$\Gamma_{a_2\to\bar{K}K}$	5.2 ± 1.1	6.64	1.716
$\Gamma_{a_2\to\eta\pi}$	15.5 ± 2.0	18.42	2.134
$\Gamma_{a_2\to\eta'\pi}$	0.57 ± 0.12	0.80	3.652
$\Gamma_{K*_2\to\bar{K}K}$	49.1 ± 2.5	40.08	~ 0
χ^2_{tot}	-	-	18.496

Mesons and Glueballs in holographic approach

- AdS/CFT correspondence [Maldacena] - equivalence between string theory defined on AdS $\times$ S^5 and QFT defined on conformal boundary of AdS space

- AdS/QCD - QCD in terms of dual string theory following AdS/CFT correspondence

- Soft-wall model [Karch et al] $S = \int dz d^4x \sqrt{g} e^{-B(z)} \mathcal{L}(z,x)$

- $g = |\mathrm{det} g_{MN}|; \qquad ds^2 = e^{2A(z)}(-dz^2 + g_{\mu\nu} dx^\mu dx^\nu)$

- Background fields $A(z)$ [metric factor] and $B(z)$ [dilaton] should obey to reproduce Regge behavior of hadron mass spectrum

- Simplest solution: $B - A \sim z^2 +$ log(z)

- E.g. $\quad A = -$log(z/R), $B = \kappa^2 z^2$,
  where κ is dilaton scale parameter, R is AdS curvature radius

- Conformal limit: $B - A \sim$ log(z) at small z

- Hard-wall model [Polchinski, Strassler]: no dilaton
  $e^{-B(z)} \to \theta(z_{\mathrm{IR}})$ [analog of MIT bag]; $\quad z_{\mathrm{IR}} \simeq$ 1/(323 MeV) is fixed from M_ρ.

Mesons and Glueballs in holographic approach

- For scalar field $\mathcal{L} = \frac{1}{2}\Big(\nabla_N \phi \nabla^N \phi + m_5^2 \phi^2\Big)$ with $m_5^2(z) = \Delta(\Delta - 4) = -3$

- $\phi(z,x) = e^{-iPx}\phi(z)$ with $P^2 = M^2$ (M is hadron mass)

- Schrödinger equation for AdS mode propagating in z-direction using substitution
  $\phi(z) = z^{3/2} e^{-\frac{1}{2}\kappa^2 z^2} \Phi(z)$ with $\int\limits_0^1 dz \Phi^2(z) = 1$

$$\Big(- \frac{d^2}{d^2 z} + \frac{3}{4z^2} + U_{\rm cf}(z) \Big) \Phi(z) = M^2 \Phi(z) \,, \quad U_{\rm cf}(z) = \kappa^4 z^2$$

- Extension to the states with any S, L and n [Karch et al; Brodsky, Téramond]

$$U_{\rm cf}(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1) \,, \quad M^2 = 4\kappa^2 \Big(n + L + \frac{S}{2} \Big)$$

- Analytic solution: $\Phi(z) = \sqrt{\frac{2n!}{(n+L)!}}\, \kappa^{1+L}\, z^{\frac{1}{2}+L}\, e^{-\frac{1}{2}\kappa^2 z^2}\, L_n^L(\kappa^2 z^2)$,
  $$M^2 = 4\kappa^2 \Big(n + L + \tfrac{S}{2} \Big)$$

Mesons and Glueballs in holographic approach

- Extension to exotic states: glueballs, hybrids, ... [Vega, Schmidt]

$$U_{\rm cf}(z) = \kappa^4 z^2 + 2\kappa^2 \Big(L + v - \frac{1}{2}\Big)\,, \qquad M^2 = 4\kappa^2 \Big(n + L + v\Big)$$

where $v = \Delta_0 + \frac{\beta}{2} - S - 1$; $\beta = -3$ for $J = 0$ and -1 for $J = 1$.

- Δ_0 is contribution to dimension of the operator creating hadrons [Vega, Schmidt, PRD79 (2009) 055003]

- Exotic states with $n = L = 0$ (left table $S = 0$; right table $S = 1$)

Δ_0	$(\bar{q}q)^m G^n$	M (GeV)	Δ_0	$(\bar{q}q)^m G^n$	M (GeV)
4	$2G$	1.28			
5	$\bar{q}qG$	1.66	5	$\bar{q}qG$	1.66
6	$(\bar{q}q)^2$	1.96	6	$(\bar{q}q)^2$ or G^3	1.96
7	$\bar{q}qG^2$	2.22	7	$\bar{q}qG^2$	2.22
8	$(\bar{q}q)^2G$ or G^4	2.46	8	$(\bar{q}q)^2G$	2.46
9	$(\bar{q}q)^3$ or $(\bar{q}q)G^3$	2.67	9	$(\bar{q}q)^3$ or $(\bar{q}q)G^3$	2.67
10	$(\bar{q}q)^2G^2$	2.87	10	$(\bar{q}q)^2G^2$ or G^5	2.87

- Quarkonia–glueball mixing (in progress)

Mesons and Glueballs in holographic approach

- Light–front holographic QCD [Brodsky, Téramond]

- Two parton case: $q_1\bar{q}_2$ mesons

$$z \to \zeta, \qquad \zeta^2 = \mathbf{b}_\perp^2 x(1-x)$$

  where ζ - impact variable; $\mathbf{b}_\perp$ - impact separation (conjugate to $\mathbf{k}_\perp$)

- Matching of AdS mode and light–front hadron wave function

$$|\psi_{q_1\bar{q}_2}(x,\zeta)|^2 = x(1-x)f^2(x)\frac{|\Phi(\zeta)|^2}{2\pi\zeta}$$

- Longitudinal mode $f(x) = 1$ or $\sqrt{2x}$ normalized as $\int\limits_0^1 dx f^2(x) = 1$

Mesons and Glueballs in chiral approach

- Extension to massive quarks [Brodsky, Téramond; Vega et al]

$$-\frac{d^2}{d\zeta^2} \to -\frac{d^2}{d\zeta^2} + \mu_{12}^2\,, \quad \mu_{12}^2 = \frac{m_1^2}{x} + \frac{m_2^2}{1-x}$$

$$f_i(x) \to f_i(x, m_1, m_2) \equiv N_i\, f_i(x)\, e^{-\dfrac{\mu_{12}^2}{2\lambda^2}}\,,$$

  where N - normalization constant, λ - additional scale parameter (Brodsky-Téramond: $\lambda \equiv \kappa$)

- New wave function and mass spectrum [Brodsky and Téramond; Lyubovitskij et al]

$$\psi_{q_1 \bar{q}_2}(x, \zeta, m_1, m_2) = \frac{\Phi(\zeta)}{\sqrt{2\pi\zeta}}\, f(x, m_1, m_2)\, \sqrt{x(1-x)}\,,$$

$$M^2 = \int\limits_0^\infty d\zeta \Phi(\zeta)\Big(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_{\rm cf}(\zeta)\Big)\Phi(\zeta) + \int\limits_0^1 dx \Big(\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\Big) f^2(x, m_1, m_2)$$

Mesons and Glueballs in chiral approach

- Modication of potential $U_{\rm cf} \to U = U_{\rm cf} + \underbrace{U_\chi}_{\rm chiral} + \underbrace{U_{\rm C}}_{\rm Coulomb} + \cdots$ (in progress)

- $U_\chi(z) = -\alpha z^2(m_1\Sigma_1 + m_2\Sigma_2) + \cdots\,, \quad 1/\alpha = F^2 \int d\zeta \zeta^2 |\Phi(\zeta)|^2$
where F - leptonic decay constant; $\Sigma_i = \langle 0|\bar q_i q_i|0\rangle$ - quark condensate

- Masses of Goldstone mesons π, K, η

$$M_\pi^2 = 2mB + m^2 \int_0^1 \frac{dx}{x(1-x)} |f(x,m,m)|^2 + \cdots$$

$$M_K^2 = (m+m_s)B + \int_0^1 dx \left(\frac{m^2}{x} + \frac{m_s^2}{1-x}\right) |f_i(x,m,m_s)|^2 + \cdots$$

$$M_\eta^2 = \frac{2}{3}(m+2m_s)B + \frac{1}{3}\int_0^1 \frac{dx}{x(1-x)} \left(m^2|f_i(x,m,m)|^2 + 2m_s^2|f_i(x,m_s,m_s)|^2\right) + \cdots$$

Mesons and Glueballs in chiral approach

- Parameters:

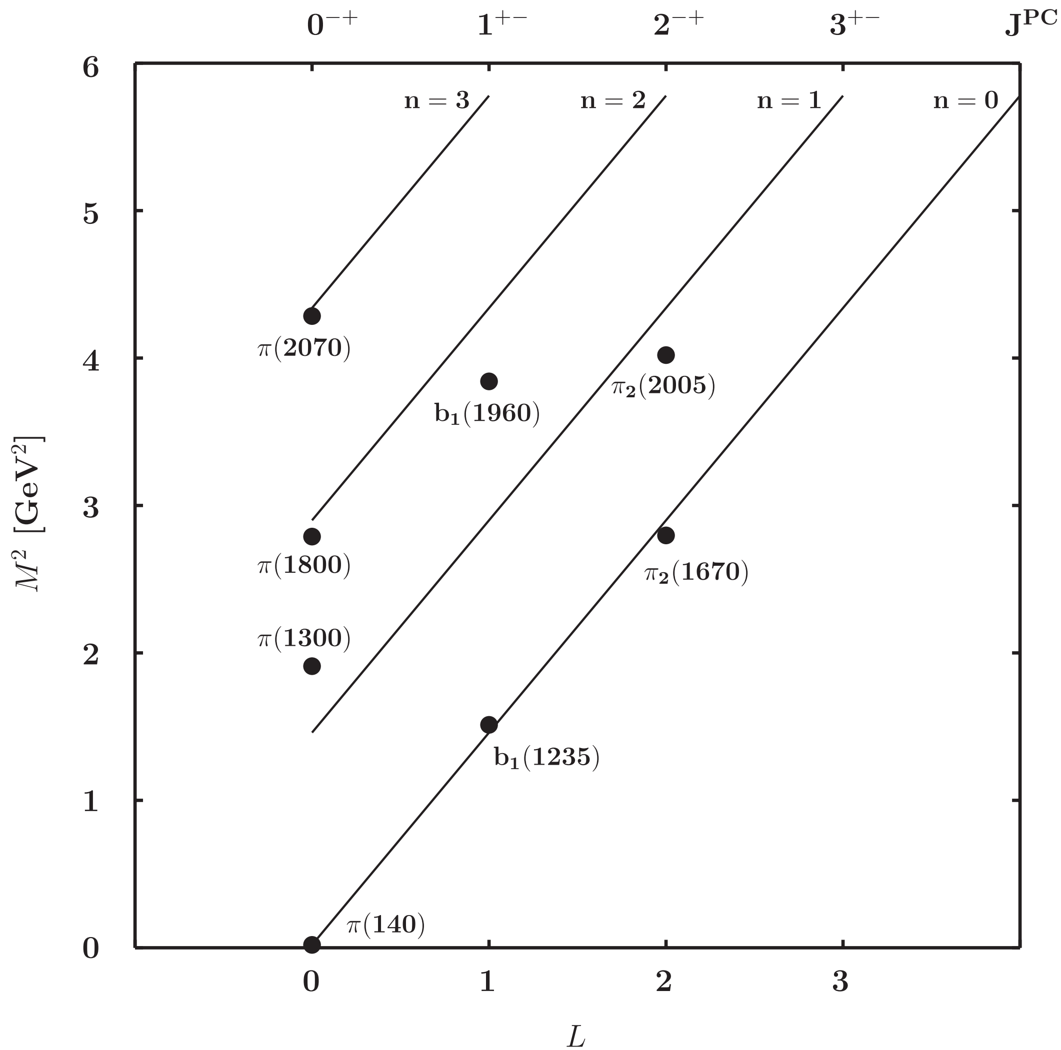
  $m_u = m_d = 7$ MeV, $m_s = 125$ MeV, $m_c = 1.27$ GeV, $m_b = 4.2$ GeV

  $\kappa = 0.6$ GeV (light and heavy-light); 1 GeV (charmonia); 1.4 GeV (bottomia)

  $\lambda = 0.6$ GeV (light); 2.7 GeV (charm); 4.8 GeV (bottom);
  2 GeV (charmonia); 7.5 GeV (bottomia)

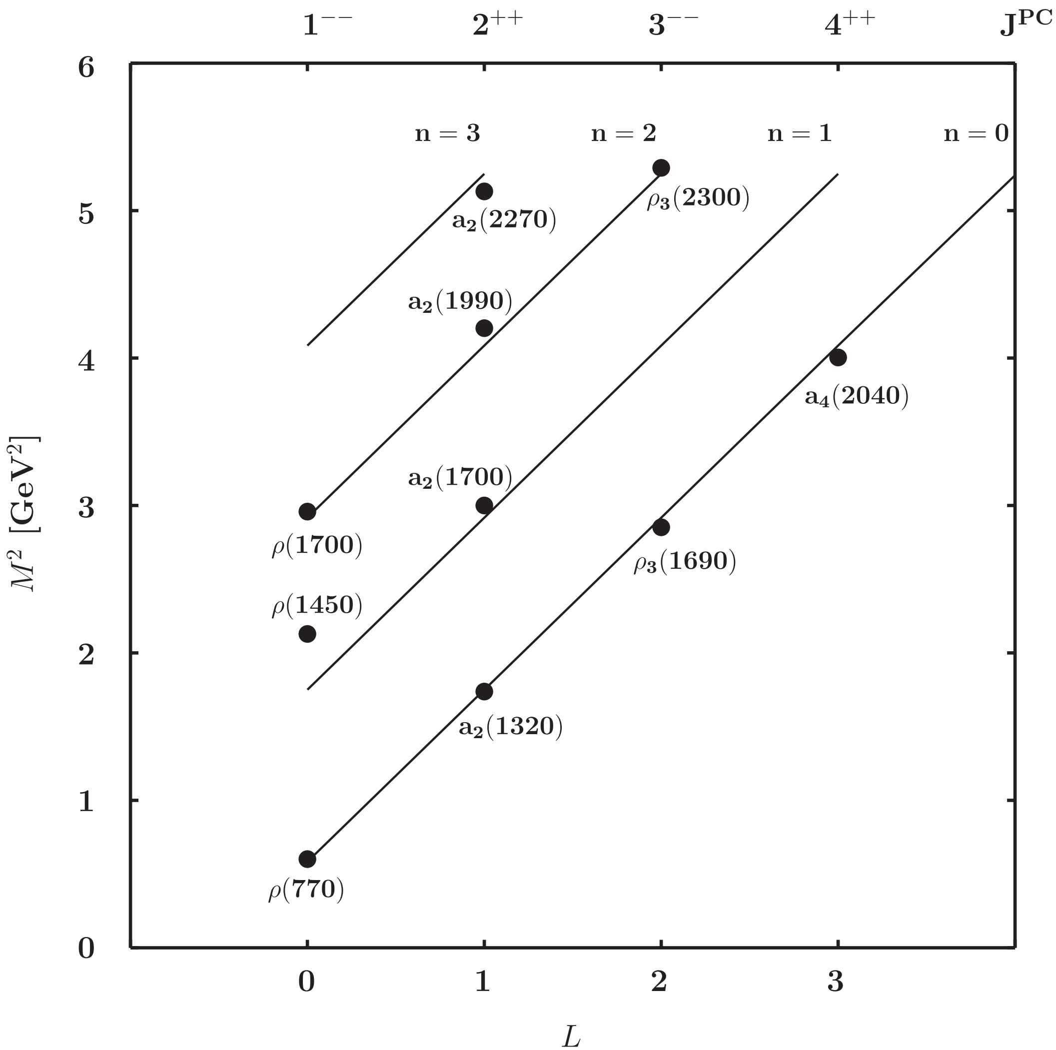
Mesons and Glueballs in chiral approach

- π family



Mesons and Glueballs in chiral approach

- ρ family



Mesons and Glueballs in chiral approach

- Heavy–light mesons

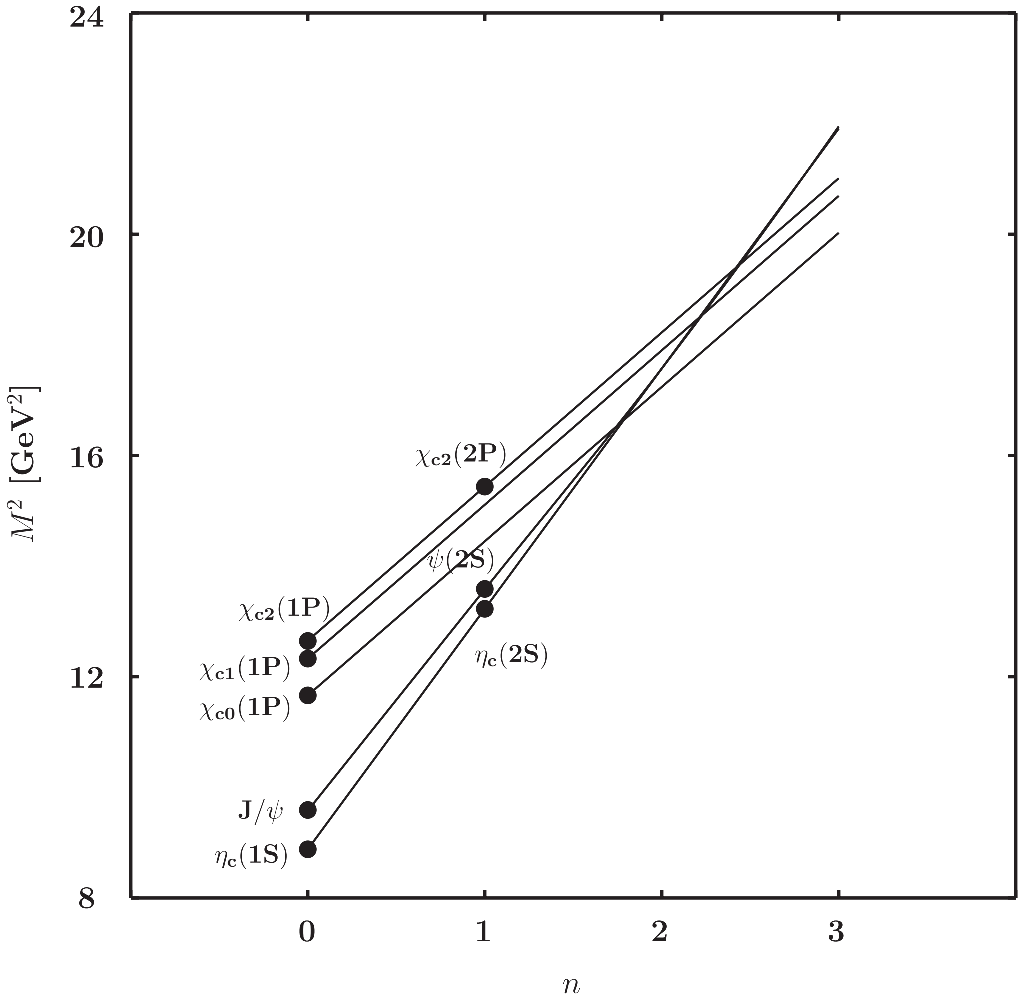
$$
\begin{aligned}
M_{Qq}^2 &= 4\kappa^2\left(n+L+\frac{S}{2}\right)+m_q B+m_Q B_Q+\int\limits_0^1 dx\left(\frac{m_q^2}{x}+\frac{m_Q^2}{1-x}\right)f^2(x,m_q,m_Q)+\cdots \\
&= \left(m_Q+\bar{\Lambda}+\mathcal{O}(1/m_Q)\right)^2
\end{aligned}
$$

- Scaling of dimensional parameters $\kappa=\mathcal{O}(m_Q^0)$, $\lambda=\mathcal{O}(\sqrt{m_Q})$
- Agreement with HQET: $M_V-M_P=\dfrac{2\kappa^2}{M_V+M_P}\sim\dfrac{1}{m_Q}$
- Heavy quarkonia

$$
M_{Q_1\bar{Q}_2}=m_{Q_1}+m_{Q_2}+E+\mathcal{O}(1/m_{Q_{1,2}})
$$

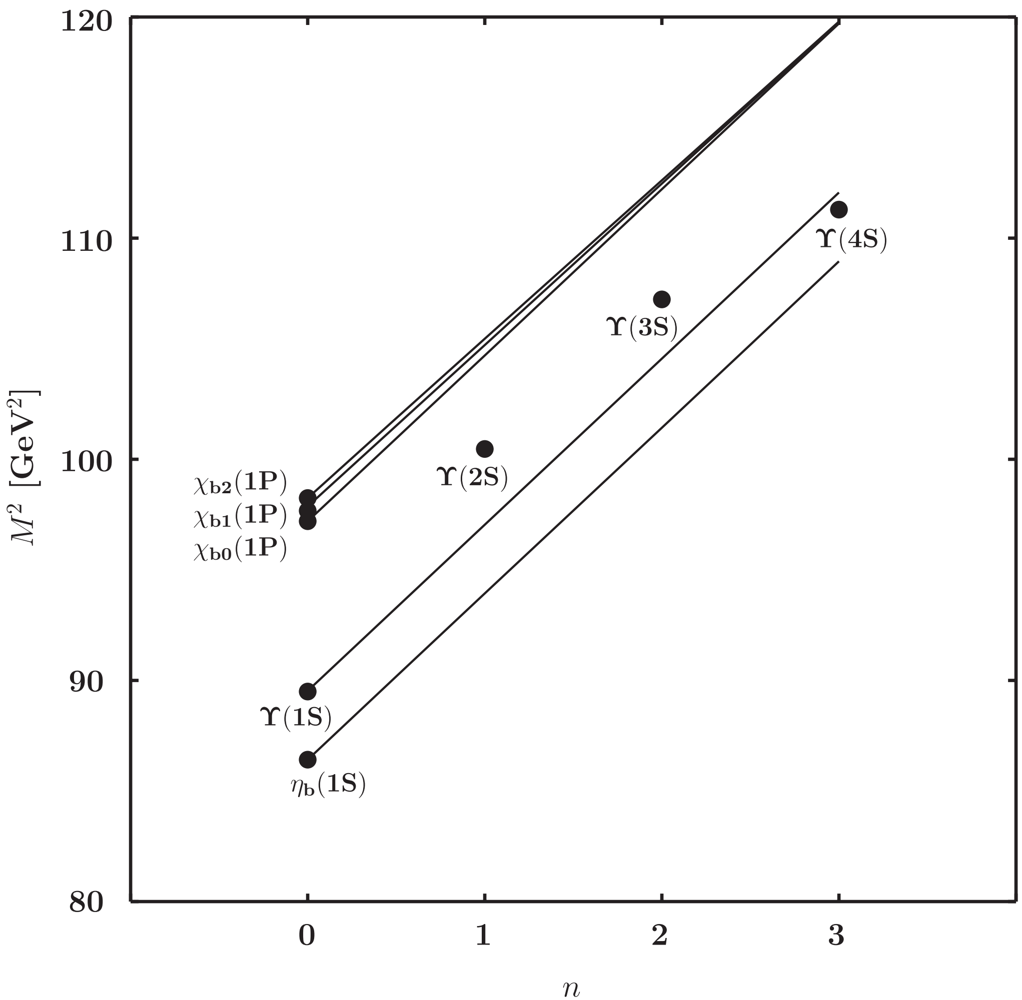
Mesons and Glueballs in chiral approach

- Charmonium spectrum



Mesons and Glueballs in chiral approach

- Bottomia spectrum



Mesons and Glueballs in chiral approach

- Decay constants f_P of pseudoscalar mesons

Meson	Data [MeV]	f_P [MeV]	$R = f_P/f_P^{\rm exp}$	κ [GeV]
π^-	$130.4 \pm 0.04 \pm 0.2$	183.4 (130.4)	1.41 (1)	0.6 (0.425)
K^-	$155.5 \pm 0.2 \pm 0.8$	186.1 (155.5)	1.20 (1)	0.6 (0.507)
D^+	205.8 ± 8.9	182.1	0.89	0.6
D_s^+	273 ± 10	183	0.67	0.6
B^-	216 ± 22	165.7	0.77	0.6
B_s^0	$253 \pm 8 \pm 7$	166.2	0.66	0.6
B_c	$399, 489 \pm 5 \pm 3$	399	1	1.33

- Decay constants f_V of vector mesons with open flavor

Meson	Data [MeV]	f_V [MeV]	$R = f_V/f_V^{\rm exp}$	κ [GeV]
D^*	$245 \pm 20^{+3}_{-2}$	182.1	0.74	0.6
D_s^*	$272 \pm 16^{+3}_{-20}$	183	0.67	0.6
B^*	$196 \pm 24^{+39}_{-2}$	165.7	0.85	0.6
B_s^*	$229 \pm 20^{+41}_{-16}$	166.2	0.73	0.6

- Decay constants f_V of vector mesons with hidden flavor

Meson	Data [MeV]	f_V [MeV]	$R = f_V/f_V^{\rm exp}$	κ [GeV]
ρ^0	154.7	130	0.84	0.6
ω	45.8	43.3	0.95	0.6
ϕ	76	63.2	0.83	0.6
J/ψ	277.6	201.1	0.72	1
$\Upsilon(1s)$	238.4	142.2	0.60	1.37

Summary

- Effective chiral Lagrangians – phenomenological tool to extract possible glueball-quarkonia mixing scenarios from observed decays; works well for tensor mesons
- Description of and predictions for strong and radiative decays
- LFH approach is a covariant and analytic model for hadron structure with confinement at large distances and conformal behavior at short distances
- It is analogous to the Schrödinger theory for atomic physics and provides the precise mapping of string modes $\Phi(z)$ in the AdS fifth dimension z to the hadron light–front wave functions in physical space-time in terms of light–front impact variable ζ, which measures the separation of the quark and gluonic constituents inside a hadron
- Mass spectrum and decay constants
- Future work:

  Extension to baryons, quarkonia–glueball mixing, hadron form factors